To Disclose or Not to Disclose: Cheap Talk with Uncertain Biases*

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Abstract. I study strategic information transmission when biases are uncertain. A perfectly informed expert advises a decision maker. The expert has biases with direction unknown to the decision maker. I show that all equilibria are of partitional form as identified by Crawford and Sobel (1982). It never benefits the decision maker or the expert to have the bias of the expert disclosed. The decision maker is better off when the bias distribution is more balanced or when the bias size is smaller.

Keywords: Cheap Talk, Transparency, Advice.

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1 Introduction

> One can only give an unbiased opinion about things that do not interest one, which is no doubt the reason an unbiased opinion is always valueless. The man who sees both sides of a question is a man who sees absolutely nothing.
>
> Oscar Wilde (1856-1900)

People frequently must make decisions without full information. Investors have to decide whether to buy or sell a stock. Moviegoers have to decide whether to purchase a ticket and see a movie. Legislators have to decide on how to vote on bills. CEO's have to make budgetary and personnel decisions. In all these cases, decision makers do not themselves have the amount of information they would ideally like to have. Consequently, they often obtain advice from experts who are better informed.

Experts are rarely indifferent about the final action taken by the decision maker. Nor is it often the case that experts have exactly the same objectives as decision makers. An investment advisor or analyst may have taken a long or short position on the stock about which she makes recommendations.[1] A movie critic may be harsh or nice towards a genre of movies. A political advisor may have conservative or liberal biases, or be beholden to special interests. Therefore, experts may have incentives to distort information so as to induce decisions that are favorable to them. Decision makers are aware that experts may be biased, and so must account for this when making inferences from experts' reports. Even when they have different objectives, experts and decision makers have a mutual interest in achieving communication in many cases, since mistakes made by the decision maker are also potentially costly to the expert.

Often, decision makers are uncertain about experts' biases. Experts may make conscious efforts to hide their biases. Even when information about biases is available, it may be difficult to acquire. For example, even when an analyst does not actively hide her conflict of interest in a stock, it may take a lot of effort for an investor to discover whether a conflict exists. A moviegoer may have to look through a number of past reviews of movies he has seen to discover a movie critic's bias towards a particular genre of movies. The benefits of obtaining such information may well be overwhelmed by the costs.

[1]For pronoun consistency, the expert in this paper is referred to as "she" and the decision maker "he."

Transparency is one of the key issues examined in this paper. When conflicts of interest arise, does transparency about biases improve communication? Should disclosure of such conflicts be mandated, or should those with conflicts of interest be required to recuse themselves?

In this paper, I model a situation in which an expert provides advice to a decision maker. The decision maker is uncertain about the final outcome of his action, which depends on some underlying state. The expert does know the state, but her preferences regarding the outcome are different from those of the decision maker. The expert's bias is her private information.[2] This model is an extension of the basic cheap talk model studied by Crawford and Sobel (1982).

I fully characterize the equilibria of the game and prove two main results. First, disclosure of biases never improves communication. That is, neither the expert nor the decision maker prefers to have the bias of the expert disclosed.

When the expert is biased and her bias is common knowledge, the difficulty in communication arises because the expert wants to make a report that favors her interests. Knowing this, the decision maker must adjust his decisions appropriately. If the expert makes a report that is too indicative of the underlying state, the decision maker's action is far from the expert's favorite. The expert would then prefer to send a report that induces some other action. This hinders effective communication.

Now suppose that the expert could have either a positive bias or a negative bias, and that the direction is unknown to the decision maker. Thus, the same report could be sent in a lower state by the expert when she has a positive bias or in a higher state by the expert when she has a negative bias. In equilibrium, the decision maker takes an action that is adjusted from the report according to the *expected* misstatement, reducing the tension between the expert and the decision maker, and rendering the expert's report more indicative of the underlying state. This benefits the decision maker, since it reduces the variance of the deviation of actions from the real states. When disclosure policies force experts to reveal their biases, this effect vanishes. This is why disclosure does not achieve improvements in communication.

When the expert's bias is known to the decision maker, since the decision maker is rational, the expert cannot lead him to choose actions that systematically deviate from the true state. She can only increase her utility by reducing the variance of the deviation of the decision maker's actions from the real states. However, her ability to do so is restricted due to her bias. When the expert's bias is not known to the decision

[2] I use the word "bias" and the phrase "conflict of interest" interchangeably to describe preference misalignments between the decision maker and the expert.

maker, on average, the expert still cannot make the decision maker choose actions that systematically deviate from the true state. However, she can make overstatements when she holds a positive bias, and make understatements when she holds a negative bias. Therefore, the decision maker's actions are closer to the expert's most preferred actions when biases are unknown to the decision maker. On the other hand, the expert can also reduce the variance of the deviation of the decision maker's actions from the real states. Thus, her expected payoff is higher when her bias is unknown to the decision maker. The expert does not want her bias revealed.

Second, the more balanced the distribution of the expert's bias, or the smaller its size, the better off is the decision maker.

As the distribution of the bias becomes more balanced, or as the size of the bias becomes smaller, the tension between the decision maker and the expert becomes weaker. This makes effective communication more likely between the expert and the decision maker.

The rest of the paper is organized as follows. In Section 2, I provide examples in which uncertainty about biases is a concern and review existing literature. In Section 3, I develop a simple model of cheap talk with uncertainty about biases. In Section 4, I fully characterize the equilibria of the cheap talk game. In Section 5, I investigate how the decision maker's payoff depends on the distribution of biases, and determine the effect of mandatory disclosure policies. Finally, in Section 6, I interpret the results, relate them to examples and the literature, and suggest directions for further research. All proofs that are not in the main text are collected in the Appendix.

2 Motivating Examples and Literature

In this section, I describe situations in which uncertainty about biases is a concern.I also discuss the existing literature and compare the differences between it and my work.

Examples. In professions that provide advice to customers and audiences, there are many rules and regulations that deal with conflicts of interest. Take, for example, business journalism. News organizations adopt two kinds of rules regarding conflicts of interest. The first kind requires that reporters avoid conflicts of interest altogether or recuse themselves when such conflicts are present. The Wall Street Journal, CBS Marketwatch, and TheStreet.com use rules of this type. The second type of rules require only that such conflicts be fully disclosed. CNBC and Motley Fools have rules like this.[3] Maria Bartiromo, the stock-market reporter and anchorwoman for CNBC, recently caused controversy when she revealed that she was holding Citigroup stocks before interviewing the company's resigning CEO, Sanford I. Weill. Amy Zelvin, a spokeswoman for CNBC, said that Ms. Bartiromo had abided by the network's policies, which require disclosure of stock ownership during on-air discussions about companies that involve more than passing mention. On the other hand, Robert M. Steele, senior faculty member and ethics group leader at the Poynter Institute, said, "Disclosure doesn't resolve a conflict of interest; all it does is reveal that a conflict exists."[4] Do disclosure policies serve as remedy for presence of conflicts of interest? Or should conflicts of interest be avoided altogether? This paper sheds some light on the merits of each side of this debate.

While news organizations strive to avoid biases or to make their employees' biases transparent, many lobbying organizations strive to hide theirs. They commonly use misleading names or mission statements to conceal their true interests, and to present themselves as educational, academic, and nonpartisan organizations.[5] Water Environment Federation, "although its name evokes images of cascading mountain streams," is the sewage industry's main trade, lobby, and public relations organization.[6] American Council on Science and Health is a group partially funded by corporations like Anheuser-Busch, Giba-Geigy, Dow Chemical, etc. Its members frequently publish articles and write op-ed pieces to refute charges of cancer risks from chemicals and food additives.[7] The general public is often misinformed about the real interests of such organizations. An interesting question to ask in this case is: should

[3]It should be noted that when a news organization decides on these policies, it also has to take into account the effect they have on its hiring practices. Having stringent policies on conflicts of interest may help enhance an organization's image among its readers or audience, but it may also negatively affects the news organization's ability to attract talented reporters, which indirectly hurts its ability to provide higher quality reports.

[4]This example is taken from McGeehan (2003).

[5]News organizations and lobbying organizations are different entities. The former attract audiences or readers at least partially through the informativeness of their news reports. They may therefore optimally choose their bias to enhance credibility. On the other hand, lobbying organizations are intrinsically biased. It is then natural that they try to increase their influence by hiding their biases.

[6]The quote comes from Stauber and Rampton (1995). Related information can be found at the organization's web site: http://www.wef.org/. The web site states the organization's mission as "preserving and enhancing the global water environment."

[7]See Lutz (1996), Chapter 6, page 175. A search through Lexis-Nexis will turn up many op-ed pieces and letters to the editor written by members of the organization, with a detectable common theme.

these organizations be required to disclose their interests in unequivocal language? Should newspapers add a disclaimer about conflicts of interest whenever they publish an article or letter written by a member of a special interest group?

Literature. The model presented in this paper is closely related to the model of Morgan and Stocken (2003) and somewhat related to that of Morris (2001). The former model incorporates a continuous state space, and assumes that the expert has perfect information about the state, as the model in this paper does. The latter employs a discrete state space, and assumes that the expert has imperfect information about the state. The similarity between their models and my own is that the decision maker is unsure of the expert's bias, and that the expert behaves strategically regardless of her bias. However, the bias distribution is skewed in one direction in their models – there are "good" advisors who are unbiased and "bad" advisors who have a non-zero bias in one direction. Thus, "bad" advisors always hurt "good" advisors' abilities to effectively communicate to the decision maker. However, models with a skewed bias distribution fail to capture the mitigating effect of the existence of opposite biases which is the focus of this paper.

From a purely theoretical point of view, this paper extends Crawford and Sobel (1982) (CS henceforth) to study cheap talk when the the expert's bias is uncertain. Unlike Morgan and Stocken (2003), I allow uncertainty about the direction of the expert's bias. I also perform a comparative static analysis; this has not been done for cheap talk models with uncertain biases. It is worth pointing out that this paper benefits significantly from CS and Morgan and Stocken (2003) in its methods of proof.

Bénabou and Laroque (1992) and Sobel (1985) also consider uncertainty about expert types, and focus on experts' reputation incentives. In their research, the "good" advisors are nonstrategic and always tell the truth, while the "bad" advisors are strategic and have incentives which directly conflict with those of the decision maker. Their models are thus very different from mine. There is also research that focuses on uncertainty about another dimension – competence of experts or accuracy of experts' information. Austen-Smith (1990), Ottaviani and Sørensen (2001), and Moscarini (2003) make contributions in this direction.

Farrell and Gibbons (1989) study the effects of the presence of different *audiences* on cheap talk, under a much different setup based on a discrete state space. They find the effect could be subversion, one-sided discipline, or mutual discipline.[8] In contrast,

[8] *Subversion* refers to cases in which the speaker is able to communicate to one audience in private, but the presence of another audience prevents him from such communication in public. *One-Sided Discipline* means that the speaker cannot communicate to one audience in private, but the presence of another audience enables him to effectively communicate with this audience. *Mutual discipline*

I study the effect of the existence of different types of *speakers*. The effect that one type of experts has on the other type is close in spirit to the "mutual discipline" effect in Farrell and Gibbons (1989).

3 The Model

An expert (E) gives advice to a decision maker (D). The decision maker makes a decision that affects both his own and the expert's payoffs. Their payoffs also depend on the value of an underlying state. The state s is a random variable uniformly distributed on $[0, 1]$. The realization of s is observable to the expert, but not to the decision maker. The expert sends a costless message m from the message set M after observing the true state. After receiving the message, the decision maker takes action $y \in \mathbf{R}$. The utility functions of the expert and the decision maker are denoted $U^E(y, s, \beta)$ and $U^D(y, s)$, where β is the expert's *bias*. They are defined by

$$\begin{aligned} U^E(y, s, \beta) &\equiv -(y - (s + \beta))^2, \\ U^D(y, s) &\equiv -(y - s)^2. \end{aligned}$$

In state s, the decision maker's most preferred action is equal to s. If the expert has bias β, her most preferred action is $s + \beta$. If $\beta > 0$, I say that the expert's bias is positive, while if $\beta < 0$, I say that the expert's bias is negative.

This model is based on the leading example in CS. Now I introduce uncertainty about biases to the model. The expert's bias is her private information, and is drawn from the following distribution:

$$\beta = \begin{cases} b & \text{with probability } p, \\ -b & \text{with probability } 1 - p. \end{cases}$$

I call the bias distribution "balanced" when p is close to $\frac{1}{2}$. When $p = \frac{1}{2}$, the expected bias of the expert is zero. A smaller b, of course, corresponds to a smaller bias size. I assume without loss of generality that $p \in [\frac{1}{2}, 1]$.

Let the message space M be $[0, 1]$. This is not a real restriction since it is rich enough for the expert to reveal all her private information. In particular, the expert may report $s/2$ when her bias is $-b$, and $(1 + s)/2$ when her bias is b.

refers to cases in which the speaker is not able to communicate to either audience in private, but is able to do so in public.

Let the joint distribution function of the message and the state be $F(s, m)$, then an agent of bias β_0 has the following expected payoff:

$$\int_{s\in[0,1]} \int_{m\in M} (y(m) - (s + \beta_0))^2 \; F(ds, dm),$$

which is equal to

$$\int_{s\in[0,1]} \int_{y\in Y} (y - (s + \beta_0))^2 \; \hat{F}(ds, dy),$$

where

$$\hat{F}(s, y) = \int_{s'\leq s} \int_{m\in M, y(m)\leq y} F(ds', dm).$$

If I combine messages that induce the same action into a single message, then the mapping from states and biases ($[0, 1] \times \{b, -b\}$) to the decision maker's actions ($[0, 1]$) are not affected. Neither is any player's payoff. In other words, this procedure simply reduces essentially equivalent strategy profiles into a single strategy profile, without affecting the number of distinct nonequivalent equilibria. From now on, I treat messages that induce the same action as one message. I will use the phrase "the message corresponding to action y" to refer to the message after receiving which the decision maker takes action y.

I consider only pure strategies for the expert.[9] A strategy for an expert with bias β can then be characterized by the function $\mu_\beta : [0, 1] \to [0, 1]$. Let $P(s|m)$ be the belief of the decision maker about the underlying state when he receives the message m. Let $y(m)$ be the action taken by the decision maker if he receives message m. Let V^D be the expected utility of the decision maker in a strategy profile. I shall later on add arguments and/or subscripts to V^D to indicate its dependence on different parameters.

The solution concept I adopt here is *Perfect Bayesian Equilibrium.* This requires that:

[9] In fact, this is without loss of generality, in terms of players' expected payoffs. Because I have reduced messages that induce the same action into one message, if an expert mixes between two reports, it must be that the two reports induce actions between which the expert is indifferent. Note that the preferences of experts guarantee that there could be at most two such actions. Also, ties between any two actions happen only in one state for an expert of any type. It is shown below that all equilibria include only a finite number of actions, and the argument does not depend on the exclusion of mixed strategies. Therefore, the points at which ties happen do not affect expected payoffs.

E1. The decision maker's beliefs, $P(\cdot|m)$, be formed using Bayes' rule for any message m whenever possible;[10]

E2. The decision maker's actions, $y(m)$, maximize his expected utility

$$\int_{[0,1]} P(s|m)U^D(y,s)\,ds$$

for all m;

E3. The expert's messages, $\mu_\beta(s)$, maximize her utility $U^E(y(m),s,\beta)$ for all s among all $m \in [0,1]$.

When describing equilibria in the rest of the paper, I omit the description of beliefs. For messages that are not sent in equilibrium, the decision maker is allowed to have any well defined belief. In particular, he can interpret all these messages as if they were one of the messages that *are* sent in equilibrium. This guarantees that the expert's incentive constraint E3 will not be violated by out-of-equilibrium messages.

As in all cheap talk models, with or without uncertainty about biases, there is always a babbling equilibrium. In such an equilibrium, there is only one equilibrium message and action. The expert sends this message regardless of the state she observes or her own bias. The decision maker takes the action $\frac{1}{2}$ no matter what message he receives. No information is transmitted in this equilibrium. However, experts and decision makers have a mutual interest in achieving *some* communication, unless their preferences are too far apart.

I use the word "informative" to describe strategy profiles that give the decision maker higher payoffs than his payoff in the babbling equilibrium. A strategy profile is "more informative" than another if the former gives the decision maker higher expected utility. In cheap talk games, it is not unreasonable to expect to observe informative equilibria when they exist. In fact, there is experimental evidence (Blume, DeJong, Kim, and Sprinkle (1998)) to support this claim. A recent paper by Kartik (2003) provides a justification in the form of an equilibrium selection criterion. Consequently, when making welfare comparisons, I focus on the most informative equilibrium.

Two useful facts. I now derive two useful consequences of the agents' quadratic utility functions.

[10] To be precise, I need $P(\cdot|m)$ to be the regular conditional probability defined by the joint distribution of m and s. This more general definition is needed for cases when the joint distribution function of m and s is neither continuous nor discrete. See Durrett (1996) for a discussion of regular conditional probabilities.

The first fact describes how an agent's ranking of two actions depends on the underlying state.[11]

Lemma 1. *For any two actions y and y', $y < y'$, an agent of bias β prefers y to y' if and only if $s \leq \frac{y+y'}{2} - \beta$.*

Proof. The state $s = \frac{y+y'}{2} - \beta$ is at the midpoint of the interval $[y - \beta, y' - \beta]$. Thus $U^E(y, s, \beta) = U^E(y', s, \beta)$ at $s = \frac{y+y'}{2} - \beta$. For any state $s \leq \frac{y+y'}{2} - \beta$, the expert's most preferred action is closer to y than y'. Lemma 1 thus follows. $\square$

Lemma 1 implies that the states in which an expert (weakly) prefers an action y to all other actions must form a closed interval when such states exist. This is because arbitrary intersections of closed intervals remain closed intervals when nonempty.

The second fact is a characterization of the decision maker's optimal action when receiving any message.

Lemma 2. *The decision maker's optimal action given any message m in equilibrium is equal to the conditional expectation $E(s|m)$.*

Proof. This Lemma can be proved by standard procedures. Any other decision rule $\tilde{y}(m)$ can be shown to add to the expected squared distance, and to reduce expected utility of the decision maker. That is,

$$\begin{aligned} E(U^D(\tilde{y}(m), s)|m) &= -E((E(s|m) - s)^2|m) - E((\tilde{y}(m) - E(s|m))^2|m) \\ &\leq E(U^D(E(s|m), s)|m). \end{aligned}$$

$\square$

Lemma 2 says that the decision maker simply takes the action that is equal to the expected value of the underlying state. In doing this, he has taken into account possible misrepresentation by the expert due to her bias.

Note that both of these facts are true regardless of the state distribution.

Before proceeding to the characterization of equilibrium, I describe what happens if the expert is required to fully disclose her bias. Later I will compare the decision maker's payoff in this case with his payoff when the decision maker is uncertain about the expert's biases.

[11] In general, Lemma 1 is true as long as the expert's preferences are strictly concave in y, $U^E_{12} > 0$, $U^E_{13} > 0$ and satisfy a symmetry condition: $U^D(y, s, \beta) = U^D(y', s', \beta')$ whenever $|y - (s + \beta)| = |y' - (s' + \beta')|$.

Full Disclosure of Biases. When the decision maker learns that the expert's bias is b, the game is identical to that in CS. CS fully characterizes the equilibria when the expert's bias is common knowledge. The set of equilibria is characterized by a partition of the interval $[0,1]$ with the boundary points $\{\alpha_i\}_{i=0}^n$ satisfying

$$\begin{aligned} \alpha_0 &= 0 \\ \alpha_{i+1} - \alpha_i &= \alpha_i - \alpha_{i-1} + 4b \quad \text{for } i = 1, 2, ..., n-1 \\ \alpha_n &= 1, \end{aligned} \tag{1}$$

for each $n \in \{1, 2, ..., N(b)\}$. I call n the *partition size*, which measures the number of partition elements in an equilibrium. The largest partition size $N(b)$ is determined by

$$N(b) = \left\lceil -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{2}{b}\right)^{1/2} \right\rceil$$

(where $\lceil z \rceil$ denotes the smallest integer larger than or equal to z). It is the largest integer n that satisfies

$$2n(n-1)b < 1.$$

In each interval (α_{i-1}, α_i) the message m_i is sent by the expert. When receiving the message m_i, the decision maker takes action

$$y_i = \frac{\alpha_{i-1} + \alpha_i}{2},$$

which is consistent with Lemma 2. At each boundary point α_i ($i = 1, ..., n-1$), the expert is indifferent between action y_i and y_{i+1}. Lemma 1 implies

$$\alpha_i = \frac{y_i + y_{i+1}}{2} - b.$$

This is how (1) is obtained. For each n, the boundary points are uniquely determined.

There exists a bijective relation between equilibria with bias b and equilibria with bias $-b$ which maps an equilibrium into its exact mirror image with respect to the point $\frac{1}{2}$. In particular, if $\{\alpha_i\}_{i=0}^n$ are the boundary points for a partition equilibrium of size n with bias b, then I may define

$$\alpha_i' = 1 - \alpha_{n-i}$$

and $\{\alpha_i'\}_{i=0}^n$ will be a partition equilibrium of size n with bias $-b$. This equilibrium gives the decision maker exactly the same expected payoff as the equilibrium with boundary points $\{\alpha_i\}_{i=0}^n$ and bias b. Furthermore, the only possible equilibria with bias $-b$ are those that can be expressed in the above form as mirror images of equilibria with bias b. Therefore, in the most informative equilibrium with bias b and in that with bias $-b$, the expected utility of the decision maker is the same. In my model with uncertainty about biases, the bias could be either b or $-b$. Therefore, when the bias of the expert is fully disclosed, the decision maker's highest utility is the same as his highest utility when $p = 1$.

It is worth noting that when biases are common knowledge, no information is transmitted when $b \geq \frac{1}{4}$. That is, $N(b) = 1$ for $b \geq \frac{1}{4}$. I will later compare this threshold with the threshold in the case where biases are unknown to the decision maker.

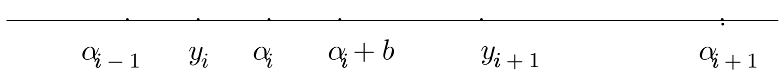

Figure 1: Part of A Partitional Equilibrium in CS

From the description of the equilibrium above, observe that if the expert has positive bias b, the lengths of partition elements are the smallest at the left end of the interval $[0, 1]$; these lengths grow as we move to the right.[12] Figure 1 illustrates the reason for this phenomenon. At the boundary point α_i, the expert is indifferent between actions y_i and y_{i+1}. But if her bias is b, her most preferred action is actually $\alpha_i + b$. Since $y_i = \frac{\alpha_i + \alpha_{i-1}}{2}$ and $y_{i+1} = \frac{\alpha_i + \alpha_{i+1}}{2}$, α_{i+1} must be shifted to the right from α_i more than the shift from α_{i-1} to α_i. This is what makes the intervals longer and communication coarser as we move along the direction of the bias. At the same time, this limits the number of messages that can be sent by the expert in equilibrium. Thus, inefficiency of information transmission occurs mainly at the end of the interval matching the direction of the expert's bias.

Let me use the metaphor of movie reviewing to illustrate this point. If a critic has a penchant for science fiction movies and her bias is known to the public, then her acclaim of "The Matrix" conveys less information than her dismissal of "Armageddon." Furthermore, since she gives the same rating to movies with varied quality levels at the high quality end, the number of possible ratings she could give to movies is also limited. Her ability to influence movie audiences' choices of science fiction movies is hurt by her bias.

This immediately brings up a possibility: when the expert can have either positive biases or negative biases, can the uneven lengths of the partition units be "av-

[12]The opposite is true if the expert has negative bias $-b$.

eraged out"? In this process, can the number of partition units be increased? Can better communication be achieved? The following two sections explore these questions.

4 Equilibria with Uncertain Biases

First I identify a necessary condition for communication to be informative in equilibrium.

Lemma 3. *There is no more than one action in equilibrium if $b \geq \frac{1}{2}$.*

Proof. Suppose there are at least two actions in equilibrium. Since $y(m) = E(s|m)$, I have $E(y(m)) = E(E(s|m)) = E(s) = \frac{1}{2}$. Let y, y' be the maximum and minimum of actions taken in equilibrium respectively.[13] It must be the case that $y < \frac{1}{2} < y'$. The action y' is the most preferred action by an expert of bias b if $s \geq \max\{y' - b, 0\}$. On the other hand, using Lemma 1, the message corresponding to action y' could only be sent by an expert of bias $-b$ if

$$\frac{y + y'}{2} + b < 1.$$

If this condition does not hold, then

$$y' \leq \frac{1 + \max\{y' - b, 0\}}{2} = \begin{cases} \frac{1}{2} & \text{if } y' \leq b; \\ \frac{y'}{2} + \frac{1-b}{2} & \text{if } y' > b. \end{cases}$$

The inequality sign is because there may exist $s \leq y' - b$ in which an expert with bias b sends the message corresponding to y'. In either case of the above equation, there is a contradiction since $b \geq \frac{1}{2}$ and $y' > \frac{1}{2}$. If the condition $\frac{y+y'}{2} + b < 1$ does hold, then since $b \geq \frac{1}{2}$, I have

$$\frac{y + y'}{2} - b < 0.$$

Thus by Lemma 1, the message corresponding to y is never sent by an expert of bias b. A contradiction similar to the one for y' above can be derived for y. So we can never have more than one action in equilibrium. □

This Lemma is a negative result regarding extremist experts: when experts' biases are large in magnitude, babbling becomes the unique equilibrium. Intuitively,

[13] When the maximum and/or the minimum do not exist, I can use the supremum and/or infimum instead. Limiting arguments can be used to reach the same conclusion by the same reasoning. This applies to below proofs as well.

since biases are large, whenever there are two actions on opposite sides of $\frac{1}{2}$, at least one type of experts strictly prefers one action y to the other action y' in all states. Thus, the message corresponding to y' is only sent by the other type. However, biases are so large that after correction, the decision maker would not want to take action y' when receiving the corresponding message. This generates a contradiction.

This result can be contrasted with results from models in which experts are either unbiased or have positive bias b. In those models, even when b is large, the unbiased expert can still reveal the value of the state, as long as the realized state is below a threshold (see Proposition 2, Morgan and Stocken (2003)). Furthermore, this threshold is independent of b as long as b is large enough, and converges to 1 as the probability of the expert's being biased converges to zero. Thus the existence of extremist experts does not hurt communication much, as long as they are relatively rare. This is true because an expert with a large positive bias does not want to send a message that induces a relatively low action. Thus, an unbiased expert can reveal low states. However, the above Lemma shows that if there are only extremists with positive biases and extremists with negative biases, then large biases preclude communication.

On the other hand, the threshold, $\frac{1}{2}$, is larger than that in the full disclosure case, $\frac{1}{4}$. In fact, as long as $b < \frac{1}{2}$, informative equilibria do exist for all $p \in [\frac{1}{2}, 1)$. So indeed, allowing uncertainty about biases expands the set of bias sizes for which informative communication is possible. Now I consider the special case $p = \frac{1}{2}$ to demonstrate this fact, and reserve the discussion.[14]

Lemma 4. *When $b < \frac{1}{2}$ and $p = \frac{1}{2}$, the following strategy profile constitutes an equilibrium:*

1. *An expert of bias β's strategy satisfies*

$$\mu_\beta(s) = \begin{cases} b & \text{if } \beta = -b, \text{ and } s \in [0, 2b]; \\ s + \beta & \text{if } s \in [b - \beta, 1 - b - \beta]; \\ 1 - b & \text{if } \beta = b, \text{ and } s \in [1 - 2b, 1]. \end{cases}$$

2. *Upon receiving message m, the decision maker takes action $y(m) = m$ for all $m \in [b, 1 - b]$. For any other message, $y(m)$ can be any action in $[b, 1 - b]$.*

When $p = \frac{1}{2}$, the distribution of biases is balanced. For any action y in the interval $(b, 1 - b)$, the positive-biased expert sends the corresponding message when

[14] The equilibrium construction for $p = \frac{1}{2}$ also appears in de Garidel-Thoron and Ottaviani (2000).

$s = y - b$, and the negative-biased expert sends it when $s = y + b$. It is optimal for the decision maker to take action y when he receives this message, as the expected misstatement is zero due to the balanced distribution. As a result, when reports in the open interval $(b, 1-b)$ are sent, any small variation in the underlying state is reflected in the expert's report. Although these reports correspond to actions different from the true state, the variance of the distance is small.

Let me return to the movie critic metaphor. If there are some critics who dislike science fiction movies and other critics who like them, and if their biases are unknown to moviegoers, then the former will provide much information when the movie is of high or medium quality, and the latter will provide much information when the movie is of bad or medium quality. On the other hand, communication is coarse at the two ends in the direction of the expert's bias. Thus, the existence of opposite opinions makes sure that the expert only resorts to uninformative rhetoric when the state is near the extreme in the direction of her biases. Similar effects exist for all other $p \in (\frac{1}{2}, 1)$, as will be shown in the general discussion that follows.

In fact, the equilibrium constructed above is the most informative equilibrium for $p = \frac{1}{2}$ in any mechanism without monetary transfers. It has been shown that for quadratic preferences and uniform state distribution, in the most informative equilibrium of all possible mechanisms, the decision maker would commit to choosing actions in the interval $[b, 1 - b]$ when it is common knowledge that the expert's bias is b. The mapping from states to actions is exactly the same as that defined by Lemma 4. Thus his optimal expected payoff is the same as in Lemma 4, conditional on the expert's bias being b.[15] The same is true when it is common knowledge that the bias of the expert is $-b$. From the view of mechanism design, the optimal outcome in which the decision maker knows the expert's bias is necessarily at least as good as the optimal outcome in which the decision maker does not. The reason is that in the former, the decision maker can choose different incentive schemes for different biases of the expert, but in the latter, he must choose the same incentive scheme for the expert no matter what her bias is. Since the decision maker achieves the same payoff in Lemma 4 as in the case where he *does* know the expert's bias, his payoff in Lemma 4 must be the highest in the case where he *does not* know the expert's bias.

It is straightforward to show that the decision maker's expected utility in this

[15] I thank Timofey Mylovanov for pointing this out to me. The argument that this is the best outcome can be found in his paper, Mylovanov (2003).

equilibrium is

$$V^D = -(1-2b)b^2 - \int_0^{2b} (s-b)^2 ds = -b^2 + \frac{4}{3}b^3.$$

Later on, I compare this expression with payoffs from other equilibria and models.

Due to Lemma 3, there exist informative equilibria only if $b < \frac{1}{2}$. So I consider only $b \in (0, \frac{1}{2})$ throughout the rest of the paper. First, I establish a result describing the expert's behavior in equilibrium. It is assumed in Morgan and Stocken (2003) when they discuss partitional equilibria, but it is an implication of the model here.

Lemma 5. *In any informative equilibrium of the game, there is no message that is sent by only one type of expert. In other words, the decision maker can never infer with certainty the expert's bias from her reports.*

Proof. First, any message corresponding to $y \in [b, 1-b]$ cannot be sent by only one type of expert. The reason is that an expert of bias β finds the action y strictly better than all other actions at state $s = y - \beta$.

Second, there cannot be messages corresponding to actions in $[0, b) \cup (1-b, 1]$ that are sent by only one type of expert. I consider $y \in [0, b)$ only, since the case $y \in (1-b, 1]$ is similar. Let y' be the smallest such action and y'' be the largest.

I claim that $y' = y''$, which means there can be at most one action in $[0, b)$ satisfying the above requirement. If $y'' > y'$ instead, then message y' is never sent by a type b expert since y'' is strictly preferred. However a type $-b$ expert strictly prefers y' to any other action when $s \in [0, y' + b]$; thus, $y' \geq \frac{y'+b}{2}$, contradicting the assumption that $y' \in [0, b)$.

Since the equilibrium is informative, let $\hat{y}$ be the smallest action in equilibrium that is greater than y'. Now, by Lemma 1, a type $-b$ expert strictly prefers y' to all other actions when $s < \frac{y'+\hat{y}}{2} + b$. By our assumption, the message corresponding to y' is only sent by type $-b$ experts. Therefore

$$y' \geq \frac{\frac{y'+\hat{y}}{2} + b}{2},$$

which implies either

$$y' \geq \frac{y' + \hat{y}}{2},$$

or

$$y' \geq b.$$

This is impossible since it is assumed that $y' < \hat{y}$ and $y' < b$. $\square$

Now I show a result similar to Lemma 1 in CS.

Lemma 6. *If $p \in (1/2, 1]$, there is no equilibrium in which an infinite number of possible actions are taken .*

The intuition behind the above lemma is as follows. If actions are arbitrarily close to one another in equilibrium, the expert sends the message that induces action y if and only if s is arbitrarily close to $y - b$ when she has positive bias, and if and only if s is arbitrarily close to $y + b$ when she has negative bias. However, the decision maker bases his action upon the expected value of the underlying state. Since $p \neq \frac{1}{2}$, the distorting effects of the two types cannot exactly cancel each other. Thus, the expected value is different from y, which causes a contradiction. Hence, the fact $p \neq \frac{1}{2}$ precludes the possibility of an infinite number of actions. In fact, when $p = \frac{1}{2}$, there does exist an equilibrium in which there are infinitely many actions in equilibrium, as shown in Lemma 4.[16]

Lemma 6 establishes that there are only a finite number of actions in equilibrium if $p > \frac{1}{2}$ and $b > 0$. In particular, this means that there do not exist fully revealing equilibria or semi-revealing equilibria, in the latter of which an expert's induced action is strictly increasing in the underlying state for some interval of states. There can only be partitional equilibria in this game.

I list the equilibrium actions in ascending order: $\{y_1, ..., y_n\}$, and label the messages that correspond to these actions as $\{m_1, ..., m_n\}$. Let

$$a_i = \frac{y_i + y_{i+1}}{2} \qquad i = 1, ..., n-1.$$

This is the average of adjacent actions. It is useful for defining the boundary points of the partition for each type. For notational convenience I also define a_i^β as follows:

$$\begin{aligned} a_0^\beta &= 0, \\ a_i^\beta &= a_i - \beta \qquad \text{for } i = 2, ..., n-1, \\ a_n^\beta &= 1. \end{aligned} \tag{2}$$

Lemma 1 implies that the message m_i is sent if $s \in [a_{i-1}^\beta, a_i^\beta]$ for bias β. For any n, each type partitions the interval $[0, 1]$ into n elements, and sends the message m_i only in the i-th element. The decision maker forms Bayesian beliefs about the underlying state, and takes the action that is equal to the conditional expected value of the state.

[16] If I generalize the model to allow the positive bias and the negative bias to be of different magnitudes, then what matters in the proof is that the expected value of the bias is nonzero.

Such an equilibrium requires that $a_1 \geq b$ and $a_{n-1} \leq 1-b$.[17] The following theorem fully characterizes all the equilibria of the game (except the one specified in Lemma 4, which can be obtained as the limit of partitional equilibria as n goes to infinity).[18]

Theorem 1. *Suppose $p \in [\frac{1}{2}, 1]$ and $b \in (0, \frac{1}{2})$. For $p \in (\frac{1}{2}, 1]$, there exists some positive integer $N(p,b)$ such that each $n = 1, 2, ..., N(p,b)$ corresponds to exactly one partitional equilibrium, and these are the only equilibria. When $p = \frac{1}{2}$, a partitional equilibrium exists for each $n \in \mathbf{N}$.*

1. *The equilibrium of partition size n can be characterized by:*

   a. *The expert sends message m_i if $s \in [a_{i-1}^{\beta}, a_i^{\beta})$, where a_i^{β} is defined by (2).*

   b. *The decision maker takes action y_i upon receiving message m_i, where y_i is determined by*

$$\begin{aligned} y_1 &= \frac{a_1}{2} + \frac{b}{2} \cdot \frac{(1-2p)a_1 + b}{a_1 + (1-2p)b} = a_1 - \frac{1}{2}\delta(a_1, p, b) \\ y_i &= \frac{a_{i-1} + a_i}{2} + b(1-2p) \qquad i = 2, ..., n-1 \\ y_n &= \frac{a_{n-1} + 1}{2} + \frac{b}{2} \cdot \frac{(1-2p)(1-a_{n-1}) - b}{(1-a_{n-1}) - (1-2p)b} = a_{n-1} - \frac{1}{2}\delta(a_{n-1} - 1, p, b) \end{aligned} \tag{3}$$

   In the above equation,

$$\delta(a,p,b) \equiv \frac{a^2 - b^2}{a + (1-2p)b} \equiv a - b \cdot \frac{(1-2p)a + b}{a + (1-2p)b}.$$

2. *For $p \neq 1$, the number $N(p,b)$ is the largest integer n satisfying*

$$\frac{(b - 2b(n-2)^2(1-2p) - 1)^2 - b^2}{b - 2b(n-2)^2(1-2p) - 1 + b(1-2p)} - 4b(n-2)(1-2p) \leq 0, \tag{4}$$

   among all n that satisfy

$$b - 2b(n-2)^2(1-2p) < 1 - b.$$

   For $p = 1$, the number $N(1,b)$ is the largest integer satisfying

$$2n(n-1)b \leq 1.$$

[17] Lemma 11 in the Appendix shows that the former implies the latter when $p \geq \frac{1}{2}$.

[18] This Theorem is very similar to Proposition 3 in Morgan and Stocken (2003). However, there are two differences. First, the statement of my result does not need the qualification "suppose the decision maker cannot infer the expert's bias," which is a consequence of Lemma 5. Second, I show (in Lemma 6) that there cannot be any non-partitional equilibrium for $p \neq \frac{1}{2}$. These observations highlight the differences between models with a two-sided bias distribution and those with a one-sided distribution.

Theorem 1 confirms that in this model, the communication equilibrium is of partitional form as identified in CS. A positive-biased expert has boundary points that are exactly $2b$ less than those of a negative-biased expert. Equation (3) reflects the fact that the action taken by the decision maker after receiving any message is equal to the conditional expectation of s. Figure 2 illustrates the equilibrium of

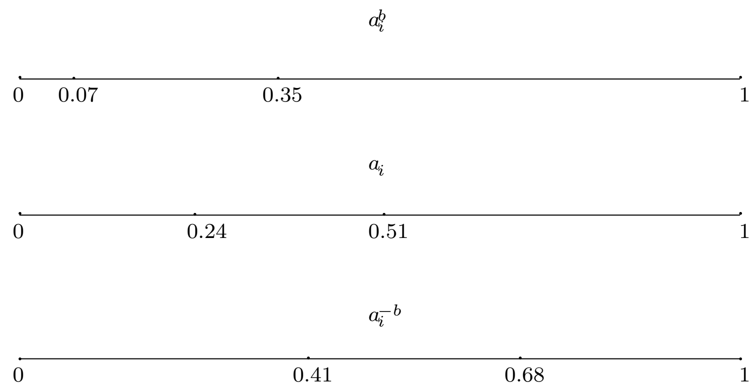

Figure 2: The size-3 equilibrium when $b = \frac{1}{6}$ and $p = \frac{2}{3}$

partition size 3 for $p = \frac{2}{3}$ and $b = \frac{1}{6}$.[19] In this equilibrium $a_1 = 0.24$ and $a_2 = 0.51$. The boundary points for an expert with bias β is a_i^β defined in (2).

Adding up adjacent equations in (3), I obtain

$$\begin{array}{rcll} a_2 - a_1 & = & \delta(a_1, p, b) - 2b(1-2p) & \\ a_i - a_{i-1} & = & a_{i-1} - a_{i-2} - 4b(1-2p) & i = 3, ..., n-1, \quad (5) \\ -\delta(a_{n-1} - 1, p, b) + 2b(1-2p) & = & a_{n-1} - a_{n-2} & \end{array}$$

The condition to determine $N(p, b)$ is obtained by setting $a_1 = b$. I need to ensure that a_{n-1} does not become too large. Suppose I consider only the first $n-2$ equations in (5). As shown in the proof of the Theorem, fixing all parameters, if the initial value a_1 increases, all boundary points will be shifted to the right, and each step $a_i - a_{i-1}$ becomes larger. However, in order for $\{a_i\}_{i=0}^n$ to be part of an equilibrium, a_{n-1} and a_{n-2} have to satisfy the last equation in (5). The left hand side is decreasing in a_{n-1} and hence decreasing in a_1, but the right hand side is increasing in a_1. Therefore, in order for there to exist an a_1 to generate boundary points that satisfy (5), the left hand side of the last equation must be greater than or equal to the right hand side when $a_1 = b$.

[19] The differences between a_2 and a_1 and those between a_2^β and a_1^β should be the same. Due to rounding, the numbers in Figure 2 do not exactly satisfy this.

Corollary 1.1. *Fix a partition size* $n \geq 2$ *and a bias value* $b \in (0, \frac{1}{2})$. *Then in equilibrium,* $\frac{\partial a_i}{\partial p} < 0$ *for* $i = 1, ..., n-1$ *at all* $p \in [\frac{1}{2}, 1]$.

The above corollary implies that all boundary points are shifted to the right as p decreases to $\frac{1}{2}$ from 1. Recall that when p is equal to one, the partition at the left end is extremely short and the partition at the right end is extremely long. One can roughly interpret this result as follows: partition elements become more even in length on average as p approaches $\frac{1}{2}$.[20] The meaning of this statement is made precise in Lemma 7 of Section 5.

Corollary 1.2. *The maximum partition size* $N(p, b)$ *is nonincreasing in* p *and nonincreasing in* b.

This means that the closer p is to $\frac{1}{2}$, the more partitional equilibria that exist. On the other hand, the smaller the size of the bias, the more partitional equilibria that exist. The latter statement generalizes a comparative static result of CS to the incomplete information setting. Now I look at two special cases.

Example 1. If n is fixed, as p approaches $\frac{1}{2}$ the left hand side of (4) converges to $\frac{(b-1)^2-b^2}{b-1}$. Since $b \in (0, \frac{1}{2})$, this expression is always negative. If I fix any $n \in N$, (4) is satisfied for p close enough $\frac{1}{2}$. Thus $N(p, b)$ goes to infinity as p approaches $\frac{1}{2}$. Also the initial point a_1 corresponding to the partition size $N(p, b)$ converges to b as p goes to $\frac{1}{2}$, since a_1 is continuous in n and p. Also, for the equilibrium of partition size $N(p, b)$, each step $a_i - a_{i-1}$ goes to zero, and a_{n-1} converges to $1 - b$. So the equilibrium with the most partition elements converges to that specified in Lemma 4.

Example 2. I now investigate the existence of informative equilibria. Such equilibria exist whenever (4) is satisfied when the number of messages n equals 2. The condition also reduces to $\frac{(b-1)^2-b^2}{b-1} \leq 0$, which is true for all $b \in (0, \frac{1}{2})$. Observe that there is a discontinuity in the existence of informative equilibria at the point $p = 1$. When $p = 1$, there are no informative equilibria for $b \geq \frac{1}{4}$. However, for all $p < 1$, there exist informative equilibria for all $b < \frac{1}{2}$.

[20] Of course, boundary points continues to shift to the right as p decreases below $\frac{1}{2}$. But now the lengths of partition elements become more uneven on average as p becomes smaller.

5 Welfare

I now study how the expected utility of the decision maker, V^D, depends on b and p in equilibrium. In a equilibrium of partition size n,

$$V_n^D(p,b) = -\sum_{i=1}^{n} p \int_{a_{i-1}^{b}}^{a_i^b} (s-y_i)^2 ds + (1-p) \int_{a_{i-1}^{-b}}^{a_i^{-b}} (s-y_i)^2 ds \tag{6}$$

First, I perform the comparative static analysis for p. The following lemma states that for any fixed partition size n, a lower p always gives the decision maker a higher expected utility.

Lemma 7. *For any fixed $b \in (0, \frac{1}{2})$ and partition size n, $V_n^D(p,b)$ is strictly decreasing in p for all $p \in (\frac{1}{2}, 1]$.*

This Lemma says that the decision maker always strictly prefers an equilibrium in which p is smaller, given any partition size n. A decrease in p has two effects on the expected utility of the decision maker. First, it decreases the weight placed on the expected utility from consulting the positive-biased type. The decision maker's expected utility from consulting the positive-biased type is lower than that from consulting the negative-biased type. Thus, the first effect increases the decision maker's expected utility. The second effect of a decrease in p is that all boundary points are shifted to the right. This decreases the decision maker's expected utility from consulting the negative-biased type but increases that from consulting the positive-biased type by the same amount. The latter carries more weight since $p \geq \frac{1}{2}$. Thus, the second effect also serves to increase the decision maker's overall expected utility. It should be noted that shifts in boundary points also lead to shifts in the decision maker's actions. But these actions are chosen by the decision maker to maximize his expected utility. The envelope theorem tells us that the marginal effect of such shifts on the decision maker's expected utility is zero. In conclusion, when the partition size n is fixed, decreasing p has the marginal effect of increasing the decision maker's expected utility.

Let $V_*^D(p,b)$ denote the decision maker's expected payoff in the most informative equilibrium of the game with prior $p \in [\frac{1}{2}, 1]$ and bias size $b \in (0, \frac{1}{2})$. Corollary 1.2 and Lemma 7 together imply the following result.

Theorem 2. *In the cheap talk game with uncertain biases, $V_*^D(p,b)$ is strictly decreasing in p. In other words, the decision maker's equilibrium utility is increasing in the symmetry of the bias distribution.*

Theorem 2 states that the more balanced the bias distribution, the more successful the information transmission from the expert to the decision maker. For intuition, first note that for any fixed partition size, Lemma 7 implies that an equilibrium with a more balanced distribution makes the decision maker better off through the two effects described above. Furthermore, with a more balanced distribution, the expected conflict of interest between the decision maker and the expert is weaker. By Corollary 1.2, this generates equilibria based on partitions with a larger number of elements. Thus, there are a larger set of equilibria from which to choose the most informative equilibrium. The decision maker's expected payoff in the most informative equilibrium becomes higher. In fact, by Lemma 8 below, an equilibrium with a larger partition size is necessarily more informative.

Lemma 8. *For any $b \in (0, \frac{1}{2})$, the decision maker always prefers an equilibrium with a larger partition size.*

This Lemma simply confirms the intuition that a larger number of partition elements makes communication more informative, and so benefits the decision maker. The proof of this Lemma is reminiscent of CS's proof of the analogous result under common knowledge of biases. The argument by CS is that an equilibrium with n partition elements can be obtained by deformation of an equilibrium with $n+1$ partition elements. During the deformation process, the decision maker's payoff continues to decrease. The process here is slightly different. The equilibrium with n partition elements cannot be obtained *exactly* by deforming the equilibrium of partition size $n+1$. Instead, I obtain a refinement of the size-n equilibrium through deformation.[21] A refinement of a partition is necessarily more informative than the original partition.[22] At the same time, the deformation process is payoff-decreasing. So the size-$n+1$ equilibrium is more informative than the size-n equilibrium.

From Lemma 8, I conclude that the most informative equilibrium for any p and b is always the one with $N(p,b)$ partition elements. That is,

$$V_*^D(p,b) = V_{N(p,b)}^D(p,b).$$

Thus for any $b \in (0, \frac{1}{2})$, the equilibrium in Lemma 4 for the case $p = \frac{1}{2}$ gives the decision maker the highest payoff among all possible equilibria for all $p \in [0,1]$.

[21] A partition A is a *refinement* of a partition A' if every partition element of A' is a disjoint union of partition elements of A. In this model, an alternative definition is that the boundary points of A' are a subset of those of A.

[22] One may appeal to the argument for Lemma 2 to prove this statement.

Furthermore, as p approaches $\frac{1}{2}$, since $N(p, b)$ goes to infinity, the most informative equilibrium approaches the equilibrium in Lemma 4.

The following Theorem is a direct implication of Theorem 2.

Theorem 3. *In the cheap talk game with uncertain biases, full disclosure of biases never benefits the decision maker.*

Proof. Note that in Section 3, I have shown that the most informative equilibrium in the case of full disclosure gives the decision maker the same payoff as that in the most informative equilibrium with $p = 1$. By Theorem 2, for any p, the decision maker's payoff in the most informative equilibrium without disclosure is always higher than the payoff under full disclosure. $\square$

Theorem 3 is a result regarding transparency. Based on the analysis of the stylized model, I draw the rather counterintuitive conclusion that transparency about biases does not improve communication. Thus, policy makers should use caution when considering policies meant to increase transparency.

I now discuss the intuition for this result. For simplicity, I use five categories to characterize the state, the decision, and the report. They could be "extremely low," "moderately low," "average," "moderately high," or "extremely high."[23] Each happens with equal probability. Also, assume a positive-biased expert would like the action to be one notch above the true state, and vice versa for a negative-biased expert. Assume also that the expert's utility is symmetric around her most preferred action.[24]

The expert is either of positive or negative bias. When the decision maker finds out what her bias is, the factors that hinder effective communication between the expert and the decision maker exist in full force. Take, for example, a positive-biased expert. At the state "moderately low," she is indifferent between actions "extremely low" and "extremely high." She cannot credibly distinguish between any of the three highest states with the other two. All these three states are thus pooled together into one message. But given the decision maker takes an action based on the expected state, the expert would also want to pool "moderately low" with the high message. The only equilibrium then involves the expert pooling the four highest states together

[23] I also allow two actions called "excessively low" and "excessively high." The former is the most preferred action by a negative-biased expert in state "extremely low." The latter is analogous. However, these are never taken by the decision maker in equilibrium.

[24] The example I consider here has a discrete space. However, the intuition discussed here is the same as that in my model with a continuous state space.

and reporting the “extremely low” state as a separate message. Much information is lost here.

However, when there is uncertainty about the expert’s bias, the decision maker can reason as follows. When the decision maker receives a moderately high message, it could be because the state is extremely high, and the expert has negative bias, or because the state is average, and the expert has positive bias. This kind of reasoning works for the middle three “moderate” messages.[25] Thus he would take the action that “matches” the meaning of the messages by taking the expected value of the state conditional on the message received. Given this reasoning by the decision maker, the expert finds it optimal to report such messages, since the action taken by the decision maker is close to her most preferred action for the bias-state combinations. This effect enables the expert and the decision maker to achieve better communication.

Now, I determine the comparative statics of welfare with respect to changes in b. First, I show that for any fixed partition size n, a lower b always gives the decision maker higher expected utility.

Lemma 9. *For any fixed p and partition size n, $V_n^D(p,b)$ is strictly decreasing in b.*

Lemma 9 says that the decision maker always strictly prefers an equilibrium in which b is smaller. The next result follows immediately from Corollary 1.2 and this Lemma.

Theorem 4. *In the cheap talk game with uncertain biases, $V_*^D(p,b)$ is strictly decreasing in b.*

This Theorem states that a smaller bias size benefits the decision maker. It confirms the analogous result in CS. The proof of Lemma 9 is an extension of the proof used in CS.

Let us now look at the problem from a different perspective. Suppose there is a population of experts. Each expert has a chance of being randomly selected to be consulted by the decision maker. I ask the question: which type of expert is made better off when the distribution is more balanced? I state the following result in a more general fashion than previous results. This theorem is true as long as both the decision maker and the expert have quadratic preferences as defined in Section 3. In particular, the result does not depend on the bias distribution or the state distribution.

[25] On the other hand, the expert still cannot separate “moderately high” and “extremely high” when she is of positive bias.

Theorem 5. *Consider any two equilibria* Γ *and* $\hat{\Gamma}$ *of a cheap talk game with uncertain biases. The corresponding expected utilities are* V^D *and* $\hat{V}^D$ *for the decision maker, and* V^{β_0} *and* $\hat{V}^{\beta_0}$ *for an expert with bias* β_0 *respectively. Then*

$$V^D \geq \hat{V}^D \Leftrightarrow V^{\beta_0} \geq \hat{V}^{\beta_0}.$$

Proof. Note each equilibrium strategy profile generates a joint distribution of s and m on $[0,1] \times \mathbf{R}$. Then

$$\begin{aligned} V^{\beta_0} &= -E(s - E(s|m) - \beta_0)^2 \\ &= -E((s - E(s|m))^2 - 2\beta_0(s - E(s|m)) + \beta_0^2) \\ &= -V^D - E(2\beta_0(E(s - E(s|m)|m))) - \beta_0^2 \\ &= V^D - \beta_0^2. \end{aligned}$$

Thus V^{β_0} differs from V^D only by a constant. Hence the desired statement. □

This Theorem implies that an expert's ranking of different equilibria is exactly the same as the decision maker's regardless of her bias. Thus, in my model, both types of experts prefer to have a more balanced distribution of biases.

Theorem 5 also implies that the expert prefers hiding her bias to disclosing it. The argument goes as follows. By revealed preferences, in each equilibrium, the expert is better off when an expert of the same type is consulted than if an expert of the other type is consulted, since the set of actions to choose from is the same in the two scenarios. Observe that her expected payoff prior to learning which type is to be consulted is a convex combination of her expected payoff from herself being consulted, and that from the other type being consulted. Therefore, her expected payoff from being consulted is higher than the expected payoff evaluated prior to learning which type of expert will be consulted. By Theorems 3 and 5, the latter payoff is higher than her expected payoff when the expert's bias is disclosed. Hence, the expert prefers not to have her bias disclosed if she knows she is the one who will be consulted.

6 Discussions

6.1 What Are the Lessons Learned?

In this paper, I consider a simple model of cheap talk in which the direction of the expert's bias is uncertain. I find that in this scenario, it is never beneficial to the decision maker or the expert to have the bias of the expert disclosed. The decision maker also benefits when the expert's bias has a more balanced distribution or a smaller size.

I now use these results to reconsider the examples in Section 2. A business journalist may have an interest in seeing the stock she covers to fall or rise. The former could be because she has taken a short position or because she wants to purchase the stock in the near future. The latter could be caused by the opposite.[26] Further suppose that the news organization mandates a policy of full disclosure. My result indicates that it is possible that such disclosure policy could actually hurt the organization's ability to convey information to its viewers, and reduce the informativeness of its reporting. Robert M. Steele's comment that disclosure does not resolve conflicts is quite relevant. The reason is that if a business journalist's conflict of interest is disclosed, her propensity to misstate and the direction of her misstatements will be common knowledge. Difficulties in communication arise, as demonstrated in Section 3. However, if uncertainty about conflicts remains, the distrust of the reporter by the viewers is likely to be mitigated. Therefore, full disclosure policies should not be viewed as a remedy measure lying somewhere "between" (1) avoiding conflicts of interest, and (2) allowing conflicts of interest and no disclosure.

Why might one expect transparency of biases to be good for communication? My opinion is that such intuition relies heavily on the assumption that decision makers may be naive, and therefore may be deceived and misled. Since all players are rational in my model, this possibility is precluded. The assessment of the relevance of this intuition depends on one's view of the decision maker's rational reasoning ability.[27]

One can draw similar conclusions regarding lobby organizations. Forcing them to disclose their true interests may not be beneficial for communication, if there are other organizations with opposing views. Theorem 5 implies that it is in lobbyists' best interests to hide their biases. There is also another important policy implication. When the debate on an issue is dominated by one side, a policy maker may want to provide support to the other side, so as to improve the public's information on the issue. For example, if industrial manufacturers fund many organizations that advocate looser product safety regulations and lower liabilities, then the government

[26]One must keep in mind that reputation incentives à la Bénabou and Laroque (1992) also exist in reality. In this paper, all players are strategic.

[27]See Crawford (2003) and Ottaviani and Squintani (2002) for interesting discussions about what happens when receivers of messages may be naive. As a reader of Dilbert, I must admit that I do not find implausible the assumption that decision makers could lack reasoning ability.

may find it beneficial to support organizations that are in favor of the opposite.[28]

At the risk of overreaching and self-promotion, I argue that this model may also imply that academic social scientists can educate the public about issues better than political pundits. The public is uncertain about the former's biases, while the latter's biases are well publicized. Consistently with what the model predicts, the former tend to use in-depth analysis and comprehensive presentation of facts, while the latter tend to resort more often to sensationalistic and uninformative rhetoric.

6.2 Multiple Experts?

In this subsection, I relate the results of this paper to those of Krishna and Morgan (2001). They show that if a decision maker sequentially consults two experts of opposite biases $b_1 < 0 < b_2$, then regardless of the size of b_1, there exists a semi-revealing equilibrium. In this equilibrium, the experts reveal the value of the state when the realization of s lies in $[0, 1 - 2b_2]$, and pool the other states together. Of course, the decision maker could also switch the order of consultation, and get all information in $[-2b_1, 1]$ revealed. Thus, the decision maker can choose to ask the less biased expert last, and obtain the most informative equilibrium.

It is straightforward to see that the equilibrium in Lemma 4 for the case $p = \frac{1}{2}$ can be generalized to the case in which $|b_1| \neq |b_2|$ and $pb_1 + (1-p)b_2 = 0$, as long as $\max\{|b_1|, |b_2|\} < \frac{1}{2}$. This equilibrium is worse for the decision maker than the Krishna and Morgan (2001) (KM henceforth) equilibrium described in the previous paragraph. First, for each bias value, what happens at the two ends of the interval (the "pooling" states) is similar to that in KM. But the decision maker's payoff in my model *is* affected by the size of both biases, while in KM it is only affected by the smaller bias. Furthermore, for states in which the expert's report is strictly increasing in the state (the "revealing" states), the decision maker's action is always $|b_i|$ away from the true state when he asks an expert of type i.

On the other hand, KM also find that sequential consultation of experts with like biases (i.e., biases in the same direction) never makes communication better than just asking the less biased expert alone. It is therefore interesting to ask the following question. Suppose that there exist both positive-biased and negative-biased experts. Two experts are randomly drawn from the population. At the beginning of the game, should the decision maker discover the biases of two experts, and then choose the best way to consult them, or should he consult them directly without discovering their biases? I now give an example to show that the latter option may be preferable.

[28]For example, the consumer advocate groups organized by trial lawyers.

Let the distribution be $(\frac{1}{2}, \frac{1}{2})$ on $\{b, -b\} = \{\frac{1}{4}, -\frac{1}{4}\}$. This bias value is chosen so that no information is revealed when each expert is consulted alone and their biases are common knowledge. If the decision maker discovers the biases of the two experts at the beginning, then according to the results of KM, his optimal decision is as follows.

1. Consult neither of them if the biases of the two experts are the same, since no information is revealed if he consults them;
2. Consult both of them if their biases are opposite to each other, in which case information is fully revealed when $s \in [0, \frac{1}{2}]$.[29]

Each case happens with probability $\frac{1}{2}$. When no information about the state is revealed, the decision maker's expected payoff is

$$-2\int_0^{\frac{1}{2}} (s - \frac{1}{2})^2 ds = -\frac{1}{12}.$$

Therefore, when the decision maker chooses to discover their biases, his payoff is

$$\frac{1}{2} \cdot (-\frac{1}{12}) + \frac{1}{2} \cdot [-(1 - 2 \times \frac{1}{4}) \times 0 - 2 \times \int_{\frac{1}{2}}^{\frac{3}{4}} (s - \frac{3}{4})^2 ds] = -\frac{3}{64}.$$

If the decision maker chooses not to discover the experts' biases and asks just one of them, then according to Lemma 4, his payoff is

$$-(\frac{1}{4})^2 + \frac{4}{3} \times (\frac{1}{4})^3 = -\frac{1}{24},$$

which is strictly higher. The decision maker's highest payoff must be at least this great if he has the option of asking them both. Thus, not knowing the experts' biases benefits the decision maker. This example points to the possibility of the existence of similar results to Theorem 3 for multiple-expert scenarios.

6.3 Communication or Delegation?

The insight provided by this model is of more significant importance when communication is the only enforceable mechanism. For example, I do not consider the possibility of monetary transfers. de Garidel-Thoron and Ottaviani (2000) show that there exist full revelation equilibria when such transfers are allowed. In addition, delegation of authority is not considered here. Dessein (2002) shows that barring money

[29] Without loss of generality, let the decision maker consult the negative-biased expert first.

transfers, when biases are common knowledge, delegation always gives the decision maker higher utility than communication, as long as communication could be informative. However, this paper illustrates that when there is uncertainty about biases, this result need not hold. In particular, full delegation gives the decision maker a payoff of $-b^2$ for sure. On the other hand, when $p = \frac{1}{2}$, the most informative communication equilibrium gives the decision maker a payoff of $-b^2 + \frac{4}{3}b^3$, which is always higher. Thus for any $b \in (0, \frac{1}{2})$, if p is close to $\frac{1}{2}$, the decision maker is better off with communication than with delegation. Therefore, although this paper is mainly concerned with scenarios in which delegation is infeasible, it also provides a reason why communication may be preferable to delegation.

In fact, a problem identified by CS is also alleviated. CS argue that if possible, under common knowledge of biases, the expert would prefer to commit to telling the truth. Without commitment, her expected payoff through communication with the decision maker is equal to $V^D - b^2$; with commitment to truth-telling, her expected payoff is the same as the decision maker's payoff when the decision maker fully delegates decisions to the expert, that is, $-b^2$. Clearly, the latter is always higher than the former since $V^D < 0$ as long as $b \neq 0$. However, when biases are uncertain, in the equilibrium of Lemma 4, the expert's payoff is

$$-\int_0^{2b} (s - (b + b))^2 \, ds + (1 - 2b) \cdot 0 = -\frac{8}{3}b^3,$$

both when her bias is b and when her bias is $-b$. Therefore, considering only $b \in (0, \frac{1}{2})$, the expert prefers to commit to truth-telling if and only if $b \geq \frac{3}{8}$. This result is also robust to small changes in p around $\frac{1}{2}$, although b may need to be smaller. Hence, when the bias is small, and when the bias distribution is balanced, the expert may prefer not to commit to telling the truth. This alleviates the problem that the expert's bias seems to be self-defeating in cheap talk models.

6.4 Generalizations and Further Research

This paper uses a stylized setup in which the state space is a bounded interval, utility functions are of the quadratic loss form, and there are only two possible bias values that are exact opposites of each other. There are various directions in which one may want to generalize the result.

This result is readily generalizable to cases where the state space is also discrete. The discussion following Theorem 3 illustrates intuitively how this can be done.

One may also want to allow other distributions of bias values. For example, one may want to allow the expert to have no bias, to have a positive or negative bias of different magnitudes, to have a bias that is always positive or negative, or to have a bias drawn from a continuum of possible values. The most informative equilibrium with complete information is known. Therefore, to prove that "nondisclosure is better than disclosure" is true, it is sufficient to identify *an* equilibrium that dominates that equilibrium. However, to prove that the statement is *not* true, it is necessary to find the most informative equilibrium under nondisclosure. The last remains an open question and is one that I am exploring. I do not know of an example in which the result is not true. However, my conjecture is that a slight variation to the setup (for example, allowing the positive bias to be of slightly different magnitude from the negative bias) does not alter "the nondisclosure dominates disclosure" result, because there is a welfare gap between nondisclosure and disclosure in the current setup, as long as there is any uncertainty.[30]

This paper compares welfare under full disclosure and that under no disclosure. It is also interesting to look at imperfect disclosure technologies.[31] For example, a technology that reveals the expert's bias correctly with some probability and wrongly the rest of the time. To see whether such imperfect technologies improve or worsen welfare, it is necessary so study the second order properties of the decision maker's utility as a function of p, the probability that the expert's bias is positive.

[30] I have been able to construct an equilibrium similar to that in Lemma 4 when negative bias and positive bias are different in magnitude but the expected bias remains zero and the bias values satisfy certain restrictions, namely, they are not too large and not too different in size. Furthermore, the equilibrium converges to that in Lemma 4 when the positive and negative biases approach each other in absolute value. Clearly when the parameter values are close to those in Lemma 4, nondisclosure dominates disclosure.

[31] A particular form of imperfect technology would not affect the result. It is clear that if the technology either discloses the expert's bias for sure or discloses nothing at all, then using this technology would make the decision maker and the expert worse off.

Appendix: Proofs

Proof. (of Lemma 4, Page 13) Given that $y(m) = m$, it is easy to see that $\mu_\beta(s) = s+\beta$ is optimal for $s \in [b-\beta, 1-b-\beta]$, since $s+\beta$ is her most preferred action. Note also this interval is neither empty nor degenerate since $b < \frac{1}{2}$ and $\beta \in \{-b, b\}$. This ensures that the profile constitutes an informative equilibrium. On the other hand, $\mu_b(s) = 1-b$ for $s \in [1-2b, 1]$ is optimal since in these states an expert with bias b prefers $1-b$ to all $y \in [b, 1-b)$ by Lemma 1. Similarly $\mu_{-b}(s) = b$ is optimal for $s \in [0, 2b]$.

Now I check the optimality of the decision maker's strategy. Given the expert's reporting strategy, the decision maker should take action $y(m) = E(s|m)$ by Lemma 2. Note

$$E(s|m) = \begin{cases} \frac{2b+0}{2} & = b \quad \text{if } m = b; \\ \frac{1}{2}(m+b) + \frac{1}{2}(m-b) & = m \quad \text{if } m \in (b, 1-b); \\ \frac{1-2b+1}{2} & = 1-b \quad \text{if } m = 1-b. \end{cases}$$

Therefore, the decision maker's strategy $y(m) = m$ for $m \in [b, 1-b]$ is optimal. Since other messages are never sent in equilibrium, he can have any belief about the underlying state. In particular, his belief can be such that $s = s_0 \in [b, 1-b]$ with probability one. Thus his optimal action when facing any other message would be $y(m) = s_0$. $\square$

Proof. (of Lemma 6, Page 16) Suppose to the contrary, there exist an infinite number of possible actions taken in equilibrium. Then by the Weierstrass-Bolzano Theorem, there exists a *monotonic* convergent sequence of actions $\{y^k\}$ with limit y.
I claim $y < b$ or $y > 1-b$ is impossible. Suppose $y < b$, then there exists K, such that $y^k < b$ for all $k \geq K$. Then in any state s, an expert of bias b strictly prefers y^K to each action y^k if $k > K$. So messages that correspond to actions $\{y^k\}_{k>K}$ can only be sent by an expert of bias $-b$. This contradicts Lemma 5. The argument is similar for $y > 1-b$.

Now consider $y \in [b, 1-b]$. Note the message that corresponds to y^k can only be sent in two scenarios: (1) the expert has bias b, and s is between $\frac{y^k+y^{k-1}}{2} - b$ and $\frac{y^k+y^{k+1}}{2} - b$; (2) the expert has bias $-b$, and s is between $\frac{y^k+y^{k-1}}{2} + b$ and $\frac{y^k+y^{k+1}}{2} + b$. I look at the case in which y^k is the only action induced in these intervals. A justification is provided in the following paragraph. As $k \to \infty$, both intervals collapse. Thus $y^k - [p(y^k - b) + (1-p)(y^k + b)]$ should converge to zero. But in fact the expression is identical to the constant $(1-2p)b$, which is negative since $p \in (\frac{1}{2}, 1]$, a contradiction. The Lemma is proved.

Now I argue that it is without loss of generality that I focus my attention on the following case: in the two intervals discussed above, y^k is assumed to be the only action the expert wants to induce. Suppose not. Then there exists another action y' which is also induced in the interval between $\frac{y^k+y^{k-1}}{2}-b$ and $\frac{y^k+y^{k+1}}{2}-b$ when the expert has bias b, which means $y' \in (y^k, y^{k+1})$. This action is also most preferred by an expert of bias $-b$ for some values of s in the interval between $\frac{y^k+y^{k-1}}{2}+b$ and $\frac{y^k+y^{k+1}}{2}+b$. I could just add this action y' to the original sequence $\{y^k\}$. I may do this as long as there are at most countably many of these y'. Call this set Y'. If Y' is uncountable, then there exist $z \in Y'$ such that for any $\varepsilon > 0$, there exist at least two other actions: $z' \in (z-\varepsilon, z)$ and $z'' \in (z, z+\varepsilon)$.[32] Thus for any $s < z-b$, there exists z' that is strictly preferred to z by an expert of bias b; and for any $s > z-b$, there exists z'' that is strictly preferred to z by an expert of bias b. A similar argument applies to an expert of bias $-b$. Thus the message corresponding to z can only be sent when the state is $z-b$ and the expert has bias b, or when the state is $z+b$ and the expert has bias $-b$. This again causes a contradiction similar to that in the previous paragraph. This argument works as long as Y' is uncountable. So Y' being uncountable is impossible. □

Proof. (of Theorem 1, Page 17) The argument preceding the theorem has shown that the only possible equilibria are partitional. Now I characterize the equilibrium.

By Lemma 1 the message m_i is sent if $s \in [a_{i-1}^{\beta}, a_i^{\beta}]$ for bias β. Let $\pi(m_i)$ be the decision maker's Bayesian belief about the probability that the expert has bias b, given the message m_i. Thus for $i = 1, ..., n$,

$$\pi(m_i) = \frac{p(a_i^b - a_{i-1}^b)}{p(a_i^b - a_{i-1}^b) + (1-p)(a_i^{-b} - a_{i-1}^{-b})}.$$

Substituting the definition of a_i^{β} into the above expression, I obtain

$$\begin{aligned} \pi(m_1) &= \frac{p(a_1 - b)}{a_1 + (1-2p)b}, \\ \pi(m_i) &= p \qquad \text{for } i = 2, ..., n-1, \\ \pi(m_n) &= \frac{p(1-(a_{n-1}-b))}{(1-a_{n-1}) - (1-2p)b}. \end{aligned}$$

Thus the partition equilibria of size n can be described by the following difference equation:

$$y_i = \pi(m_i) \cdot \frac{a_{i-1}^b + a_i^b}{2} + (1-\pi(m_i)) \cdot \frac{a_{i-1}^{-b} + a_i^{-b}}{2} \qquad \text{for } i = 1, ..., n.$$

[32] Suppose not. Then we have uncountable non-overlapping intervals summing up to finite length, which is impossible.

Substituting the expression for $\pi(m_i)$ into the above difference equation, I obtain

$$\begin{aligned} y_1 &= \frac{a_1}{2} + \frac{b}{2} \cdot \frac{(1-2p)a_1 + b}{a_1 + (1-2p)b} = a_1 - \frac{1}{2}\delta(a_1, p, b) \\ y_i &= \frac{a_{i-1} + a_i}{2} + b(1-2p) \qquad i = 2, ..., n-1 \\ y_n &= \frac{a_{n-1} + 1}{2} + \frac{b}{2} \cdot \frac{(1-2p)(1-a_{n-1}) - b}{(1-a_{n-1}) - (1-2p)b} = a_{n-1} - \frac{1}{2}\delta(a_{n-1} - 1, p, b) \end{aligned}$$

which is exactly (3). In the above difference equation,

$$\delta(a, p, b) \equiv \frac{a^2 - b^2}{a + (1-2p)b} \equiv a - b \cdot \frac{(1-2p)a + b}{a + (1-2p)b}.$$

For notational convenience, I suppress the dependence of δ on p and b when there is no confusion.

Interpretation of the Function δ. The expression $\frac{1}{2}\delta(a_1, p, b)$ can be interpreted as the downward adjustment needed to calculate y_1 given a_1. This adjustment is equal to $\alpha_1/2$ for the CS case with known bias b.[33] In my model, this effect is still present. However, an additional second effect also exists. The expert sends message m_1 more often when her bias is $-b$ ($s \in [0, a_1 + b]$) than if her bias is b ($s \in [0, a_1 - b]$). Thus the adjustment must take the expected amount of misrepresentation into account. In the definition of δ, the first term, a, accounts for the first effect. The second term, $-b \cdot \frac{(1-2p)a+b}{a+(1-2p)b}$, accounts for the second effect.

As a_1 goes up, the first term becomes larger. The reason is simple. As the upper bound gets larger, to calculate the expectation of a uniformly distributed random variable, one needs to subtract more from the upper bound. As a_1 goes up, the second term also increases. An increase in a_1 makes message m_1 relatively more likely to come from the expert when she is of positive bias b. Thus the downward adjustment needs to be larger. So δ is increasing in a_1.

As p goes up, the first effect is unchanged. However, the second term becomes larger, since an increase in p means the message m_1 is more likely to be sent by an expert with positive bias b. This increases the downward adjustment needed. So δ is increasing in b.

The effect of a change in b is ambiguous. Note that b does not appear in the first term. But an increase in b affects the second term in two ways. First, the absolute value of the adjustment associated with the second term increases. Second,

[33] This simply reflects the fact that to calculate the expected value of s given that s is uniformly distributed on $[0, \alpha_1]$, one must subtract $\alpha_1/2$ from α_1, the upper bound. Note that α_i in the CS model corresponds to $a_i^b = a_i - b$ in my model with uncertain biases.

the increase in b makes the message less likely to come from the expert when she has positive bias ($a_1 - b$ is smaller relative to $a_1 + b$). Thus the magnitude of the downward adjustment should decrease. If the second term is negative, then the two effects operate in the same direction, and an increase in b results a decrease in δ. Otherwise, the opposite could be true.

The following fact summarizes information about the derivatives of δ with respect to a, p, and b:

Fact 1. *The derivatives of function δ satisfy:*

(i) $\delta_a(\cdot) = \frac{a^2+2(1-2p)ab+b^2}{(a+(1-2p)b)^2} = 1 + \frac{(1-(1-2p)^2)b^2}{(a+(1-2p)b)^2} \geq 1$ *for* $p \in [\frac{1}{2}, 1]$.

(ii) $\delta_p(\cdot) = \frac{2b(a^2-b^2)}{(a+(1-2p)b)^2}$. *Note* $\delta_p(\cdot) \geq 0$ *if* $|a| \geq b$ *and is strictly positive if* $|(|\, a) > b$.

(iii) $\delta_b(\cdot) = \frac{-2b(a+(1-2p)b)-(1-2p)(a^2-b^2)}{(a+(1-2p)b)^2}$.

The expression $-\frac{1}{2}\delta(a_{n-1} - 1, p, b)$ is the upward adjustment needed to calculate y_n from a_{n-1} . The Fact above about derivatives of δ can similarly be used to provide an intuitive explanation for this expression.

Adding up adjacent equations in (3) and rearranging give

$$\begin{aligned} a_2 - a_1 &= \delta(a_1, p, b) - 2b(1-2p) \\ a_i - a_{i-1} &= a_{i-1} - a_{i-2} - 4b(1-2p) \qquad i = 3, ..., n-1, \\ -\delta(a_{n-1} - 1, p, b) + 2b(1-2p) &= a_{n-1} - a_{n-2} \end{aligned}$$

This is exactly Equation (5). The above difference equation can also be written in its backward form:

$$\begin{aligned} a_{n-2} - a_{n-1} &= \delta(a_{n-1} - 1, p, b) - 2b(1-2p) \\ a_{n-i} - a_{n-(i-1)} &= a_{n-(i-1)} - a_{n-(i-2)} - 4b(1-2p) \qquad i = 3, ..., n-1, \\ -\delta(a_1, p, b) + 2b(1-2p) &= a_1 - a_2 \end{aligned}$$

It is useful to look at partial solutions of (5); that is, boundary points that satisfy some equations in (5), but not the rest. The following Lemma considers boundary points that are defined by all equations except the last equation in (5) (and its backward version). It describes how they depend on the initial value a_1 (or for the backward version, a_{n-1}) and p.

Lemma 10. *In (5), consider all equations except the last one. Then $a_i - a_{i-1}$ is strictly increasing in $a_1 \geq b$ and p for $i = 2, ..., n-1$. Hence a_i is also strictly increasing in a_1 and p. Similarly, in the backward version of (5), considering all* *equations but the last,* $a_{n-i} - a_{n-(i-1)}$ *is strictly increasing in* $a_{n-1} \leq 1 - b$ *and* p *for* $i = 2, ..., n-1$. *Hence* a_{n-i} *is also strictly increasing in* a_{n-1} *and* p.

Proof. (of Lemma 10) Solving (5) forward, I get

$$a_i - a_{i-1} = \delta(a_1, \cdot) - 4b(i-2)(1-2p) - 2b(1-2p),$$

for $i = 2, ..., n-1$. By Fact 1, $\delta_a > 0$, and if $a_1 \geq b$, then $\delta_p \geq 0$. It then immediately follows that $a_i - a_{i-1}$ is strictly increasing in a_1 and p for $i = 2, ..., n-1$. Since $a_i = a_1 + \sum_{j=2} (a_j - a_{j-1})$, a_i is also strictly increasing in a_1 and p for $i = 2, ..., n-1$.

The corresponding statements for the backward version can be similarly shown.

□

Equation (5) implies

$$a_i - a_1 = (i-1)\delta(a_1, \cdot) - 2b(i-1)^2(1-2p). \tag{7}$$

The backward version of (5) gives

$$a_{n-i} - a_{n-1} = (i-1)\delta(a_{n-1} - 1, \cdot) - 2b(i-1)^2(1-2p). \tag{8}$$

Adding (7) and (8) at $i = n-1$, I have

$$\delta(a_1, \cdot) + \delta(a_{n-1} - 1, \cdot) - 4b(n-2)(1-2p) = 0. \tag{9}$$

Define function λ as

$$\lambda(a, p, b, i) \equiv a + (i-1)\delta(a, p, b) - 2b(i-1)^2(1-2p).$$

For notational convenience, I suppress the dependence of λ on p and b when there is no confusion.

Interpretation of λ. Given any initial value of a_1, $\lambda(a_1, i)$ gives the value of a_i according to (5) by solving it forward. On the other hand, $\lambda(a_{n-1} - 1, i)$ gives the value of a_{n-i} according to the backward version of (5), given an initial value of a_{n-1}. Observe the following facts about the derivatives of λ:

Fact 2. *The derivatives of function* λ *satisfy:*

(i) $\lambda_a(\cdot) = 1 + (i-1)\delta_a(\cdot) > 0$.

(ii) $\lambda_p(\cdot) = (i-1)\delta_p(\cdot) + 4b(i-1)^2$, *which is greater than or equal to* 0 *if* $|a| \geq b$.

(iii) $\lambda_b(\cdot) = (i-1)\delta_b(\cdot) - 2(i-1)^2(1-2p)$.

(iv) $\lambda_i(\cdot) = \delta(a, \cdot) - 4b(i-1)(1-2p)$, *and is nonnegative if* $\delta(a, \cdot) \geq 0$ *and* $p \in [\frac{1}{2}, 1]$.

The interpretation for the positive sign of the derivative of λ with respect to a is that all boundary points increase as the initial point increases. This is true for both forward and backward versions of (5).

Now using the definition of $\lambda(\cdot)$ in (9), I get

$$\delta(a_1, \cdot) + \delta(\lambda(a_1, n-1) - 1, \cdot) - 4b(n-2)(1-2p) = 0. \tag{10}$$

Similarly, using the definition of $\lambda(\cdot)$ in the backward version of (9) yields

$$\delta(\lambda(a_{n-1} - 1, n-1), \cdot) + \delta(a_{n-1} - 1, \cdot) - 4b(n-2)(1-2p) = 0. \tag{11}$$

The solution to Equation (10) in $[b, 1-b]$ is unique if it exists, because the left hand side is strictly increasing in a_1 by Facts 1 and 2.

Now I need to show that a unique partitional equilibrium exists for each $n \leq N(p, b)$. I first establish a useful lemma.

Lemma 11. *Fix $b > 0$, $p \in [\frac{1}{2}, 1]$, and $n \geq 2$. For any $a_1 \geq b$, if a_{n-1} satisfies (9), then $a_{n-1} \leq 1 - a_1$. Furthermore, $a_{n-1} < 1 - b$ always holds in equilibrium.*

Proof. First note that the left hand side of (9) is strictly increasing in a_{n-1} since $\delta_a(\cdot) > 0$ by Fact 1. So to prove $a_{n-1} \leq 1 - a_1$, it suffices to show that if $a_{n-1} = 1 - a_1$, then the left hand side is nonnegative.

For $p \in [\frac{1}{2}, 1]$, substituting $a_{n-1} = 1 - a_1$ into (9) yields

$$\text{L.H.S. of (9)} \quad = \quad \frac{2b(1-2p)(a_1^2 - b^2)}{(1-2p)^2 b^2 - a_1^2} - 4b(n-2)(1-2p).$$

Both terms are nonnegative since $a_1 \geq b$ and $p \in [\frac{1}{2}, 1]$.[34] Thus $a_{n-1} \leq 1 - a_1$. The first part is proved.

If $n = 2$, the above expression is equal to 0 if $a_1 = b$. This means that $a_1 = b$ and $a_{2-1} = 1 - a_1 = 1 - b$ solve (9). However, this implies $a_1 = \frac{1}{2}$, contradicting $b < \frac{1}{2}$. Thus $a_1 > b$, hence $a_{n-1} \leq 1 - a_1 < 1 - b$.

If $n \geq 3$, then the above expression is positive unless $p = \frac{1}{2}$. Thus when $p \in (\frac{1}{2}, 1]$, $a_{n-1} < 1 - a_1 \leq 1 - b$. When $p = \frac{1}{2}$, then $a_1 > b$ since otherwise $a_i - a_{i-1} = 0$ for all i, as $\delta(a_1, p, b) = 0$ at $a_1 = b$. So $a_{n-1} \leq 1 - a_1 < 1 - b$. Thus the second part is proved. □

[34] If $p = 1$, then the first term is equal to $2b$ no matter what a_1 is. In particular, when $a_1 = b$ both the denominator and the numerator are zero, but "cancelling out" $a_1^2 - b^2$ (by taking limits) gives $2b$. On the other hand, if $p = \frac{1}{2}$, then both terms are zero.

By Lemma 11, a partitional equilibrium of size n exists if and only if there exists $a_1 \geq b$ to satisfy Equation (10). We need $a_1 \geq b$ in order for $a_1^b = a_1 - b \geq 0$.

Claim. There exists $a_1 \geq b$ satisfying (10) if and only if the left hand side of (10) is nonpositive at $a_1 = b$.

Proof of Claim. Note (10) is obtained by replacing a_{n-1} with

$$\lambda(a_1, n-1) = a_1 + (n-2)\delta(a_1) - 2b(n-2)^2(1-2p)$$

in (9). As shown in the proof of Lemma 11, the left hand side of (9) is nonnegative for $a_{n-1} \geq 1 - a_1$. Note when $a_1 = \frac{1}{2}$, $\lambda(a_1, n-1) \geq a_1 = 1 - a_1$. Thus the left hand side of (10) is nonnegative when $a_1 = \frac{1}{2}$. The left hand side of (10) is a continuous and strictly increasing function of a_1. Continuity and the Intermediate Value Theorem implies there exists $a_1 \in [b, \frac{1}{2}]$ solving (10) if the left hand side of (10) is nonpositive at $a_1 = b$. On the other hand, if the left hand side of (10) is positive at $a_1 = b$, then monotonicity implies that there does not exist $a_1 \in [b, \frac{1}{2}]$ solving (10). Thus the claim is proved. $\square$

The following facts are useful:

$$\begin{array}{lcll} \delta(a,p,b) & = & a+b & \text{for } p = 1; \\ \delta(a,p,b) & = & 0 & \text{for } p \in [\frac{1}{2}, 1), a = b; \\ \lambda(a,p,b,i) & = & b & \text{for } p = \frac{1}{2}, a = b. \end{array}$$

Before proceeding, note that the left hand side of (10) is nondecreasing in n, since the derivative is

$$\delta_a(\lambda(a_1, n-1) - 1, \cdot)\lambda_i(a_1, n-1) - 4b(1-2p) \geq 0.$$

By Facts 1 and 2 and $p \in [\frac{1}{2}, 1]$, the above expression is nonnegative, and strictly positive unless $a_1 = b$ and $p = \frac{1}{2}$. Now I substitute $a_1 = b$ into (10) and obtain for $p \in [\frac{1}{2}, 1)$,

$$\frac{(b - 2b(n-2)^2(1-2p) - 1)^2 - b^2}{b - 2b(n-2)^2(1-2p) - 1 + b(1-2p)} - 4b(n-2)(1-2p) \leq 0.$$

Additionally, I need to guarantee that

$$b - 2b(n-2)^2(1-2p) < 1 - b,$$

by the second part of Lemma 11. For $p = \frac{1}{2}$, the two equations become

$$\frac{-2b+1}{b-1} \leq 0 \Leftrightarrow b \leq \frac{1}{2}$$

and

$$b < 1 - b,$$

which are satisfied by our assumption $b \in (0, \frac{1}{2})$. These conditions do not depend on n. So there exists a partitional equilibrium for each $n \in \mathbf{N}$ if $p = \frac{1}{2}$.

For $p = 1$, the condition is $(a_1+b)+(\lambda(a_1, n-1)-1+b)-4b(n-2)(1-2)|_{a_1=b} \leq 0$, which simplifies into

$$2n(n-1)b \leq 1.$$

Since the left hand side of (10) is nondecreasing in n, when $p \in (\frac{1}{2}, 1]$, the set of numbers that satisfy (4) is the set of integers that are smaller than the largest integer that satisfies (4). Furthermore since the derivative of the left hand side of (10) with respect to a_1 is

$$\delta_a(a_1, \cdot) + \delta_a(a_{n-1} - 1, \cdot)\lambda_a(a_1, \cdot),$$

which is strictly positive for any $a_1 > b$ by Facts 1 and 2, the a_1 satisfying (10) is unique. Therefore the partitional equilibrium of size n is also unique for each $n = 1, .., N(p, b)$. $\square$

Proof. (of Corollary 1.1, Page 19) First I establish that in equilibrium, a_1 is strictly decreasing in p when $p \in (\frac{1}{2}, 1)$. Applying the Implicit Function Theorem to equation (10), I obtain

$$\frac{\partial a_1}{\partial p} = -\frac{\delta_p(a_1, \cdot) + \delta_a(a_{n-1} - 1, \cdot)\lambda_p(a_1, \cdot) + \delta_p(a_{n-1} - 1, \cdot) + 8b(n-2)}{\delta_a(a_1, \cdot) + \delta_a(a_{n-1} - 1, \cdot)\lambda_a(a_1, \cdot)}.$$

By Facts 1 and 2 about the values of the derivatives of δ and λ, every term in the denominator and the numerator is nonnegative. The term

$$\delta_p(a_{n-1} - 1, \cdot)$$

is strictly positive due to $a_{n-1} < 1 - b$ by Lemma 11, and due to Part (ii) of Fact 1. Thus I conclude that $\frac{\partial a_1}{\partial p} < 0$ for all $p \in [\frac{1}{2}, 1]$.

Now I show that $\frac{\partial a_i}{\partial p} < 0$ for $i = 2, ..., n-1$ by induction. Let $1 \geq p > p' \geq \frac{1}{2}$. Let j be the smallest i such that $a_i(p) \geq a_i(p')$. Then $j \geq 2$ since it has been shown that a_1 is strictly decreasing in p. Now using (5), I have

$$\begin{array}{rcl} a_{j+1}(p) - a_j(p) & = & a_j(p) - a_{j-1}(p) - 4b(1-2p) \quad > \quad a_j(p) - a_{j-1}(p) - 4b(1-2p') \\ & > & a_j(p') - a_{j-1}(p') - 4b(1-2p') \quad = \quad a_{j+1}(p') - a_j(p'). \end{array}$$

The equality signs come directly from (5). The first inequality uses the fact that $-4b(1-2p) > -4b(1-2p')$. The second inequality sign is implied by $a_j(p) \geq a_j(p')$ and $a_{j-1}(p) < a_{j-1}(p')$. With similar arguments, and by induction,

$$a_i(p) - a_{i-1}(p) > a_i(p') - a_{i-1}(p')$$

for all $i = j+1, ..., n-1$. In particular,

$$a_{n-1}(p) - a_{n-2}(p) > a_{n-1}(p') - a_{n-2}(p').$$

Summing across i, I have

$$a_{n-1}(p) - a_j(p) > a_{n-1}(p') - a_j(p').$$

This implies $a_{n-1}(p) \geq a_{n-1}(p')$ since $a_j(p) \geq a_j(p')$. Thus I have

$$-\delta(a_{n-1}(p) - 1, p, b) + 2b(1-2p) < -\delta(a_{n-1}(p') - 1, p', b) + 2b(1-2p'),$$

since $\delta_a > 0$, $\delta_p > 0$ and $2b(1-2p) < 2b(1-2p')$. By (5),

$$-\delta(a_{n-1} - 1, p, b) + 2b(1-2p) = a_{n-1} - a_{n-2}$$

must hold in equilibrium.

The last three equations cause a contradiction.[35] Hence a_i is strictly decreasing in p in equilibrium. □

Proof. (of Corollary 1.2, Page 19) Let $\tilde{N}(p,b)$ be the number that satisfies

$$b - 2b(n-2)^2(1-2p) < 1 - b$$

and satisfies (4) with equality. I know it is unique from the proof of Theorem 1. It suffices to show that $\tilde{N}$ is nonincreasing in p and b, since $N(p,b)$ is the largest integer smaller than or equal to $\tilde{N}(p,b)$.

The left hand side of (10) is nondecreasing in n. The derivative is

$$\delta_a(\lambda(a_1) - 1, \cdot)\lambda_n(\cdot) - 4b(1-2p) \geq 0,$$

by Fact 1. Also note that (4) is obtained by setting $a_1 = b$ in (10). Thus

$$\frac{\partial(L.H.S. \text{ of } (4))}{\partial n} = \frac{\partial(L.H.S. \text{ of } (10))}{\partial n}|_{a_1=b} \geq 0.$$

[35]Note that the same proof tactic is used in the proof of Lemma 12 on Page 42.

Part 1. $\tilde{N}$ is nonincreasing in p.

Note the derivative of the left hand side of (10) with respect to p is

$$\delta_p(a_1,\cdot) + \delta_a(a_{n-1}-1,\cdot)\lambda_a(a_1,\cdot) + \delta_p(a_{n-1}-1,\cdot) + 8b(n-2),$$

which by Facts 1 and 2 is strictly positive for any $a_1 \geq b$ and $a_{n-1} = \lambda(a_1,\cdot) \leq 1-b$. I may use the implicit function theorem to obtain for $p \in [\frac{1}{2}, 1)$

$$\frac{\partial \tilde{N}}{\partial p} \;=\; -\frac{\partial(L.H.S.\text{ of }(10))/\partial p|_{a_1=b}}{\partial(L.H.S.\text{ of }(10))/\partial n|_{a_1=b}}.$$

The expression is negative since the denominator is nonnegative and the numerator is positive. Now I look at $p = 1$. Note in (4) as $p \to 1$, the inequality approaches

$$2(n-1)^2 b \leq 1.$$

In contrast, $N(1,b)$ is the largest integer that satisfies

$$2n(n-1)b \leq 1.$$

Therefore, $\lim_{p\to 1} N(p,b) \geq N(1,b)$. In fact, the largest difference between the two values is one, since $\lim_{p\to 1}\tilde{N}(p,b) - \tilde{N}(1,b) = \sqrt{\frac{1}{2b}} + 1 - (\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2}{b}}) < \frac{1}{2}$.
To summarize, $N(p,b)$ is nonincreasing in p for all $p \in [\frac{1}{2}, 1]$ and $b \in (0, \frac{1}{2})$.
Part 2. $\tilde{N}$ is nonincreasing in b.

That $\tilde{N}(1,b)$ is nonincreasing in b can be easily observed from the condition

$$2n(n-1)b \leq 1.$$

Now consider the case $p \neq 1$. Note that (4) is obtained by substituting $a_1 = b$ into (10). Also observe the facts $\delta(a_1,p,b) = 0$ and $\lambda(a_1,p,b,n-1) = 0 + b - 2b(n-2)^2(1-2p)$ at $a_1 = b$. Define

$$l(b) = b - 2b(n-2)^2(1-2p).$$

Then

$$l'(b) = 1 - 2b(n-2)^2(1-2p),$$

which is nonnegative for all $n \geq 2$. Also, $l(b) = l'(b)b$ since $l(b)$ is linear in b.

First, it is clear that $l(b) \leq 1-b$ is required since $l(b) = \lambda(b,p,b,n-1)$ is the value of a_{n-1} when $a_1 = b$. This requires that

$$2b - 2b(n-2)^2(1-2p) \leq 1,$$

which is more likely to be satisfied by smaller b for any $n \geq 2$.

Second, (4) can be written as

$$\frac{(l(b)-1)^2-b^2}{l(b)-1+b(1-2p)} - 4b(n-2)(1-2p) \leq 0.$$

Thus

$$\begin{aligned}
& \partial(L.H.S. \text{ of } (4))/\partial b \\
= & \tfrac{2[(l(b)-1)l'(b)-b][l(b)-1+b(1-2p)]-[(l(b)-1)^2-b^2][l'(b)+(1-2p)]}{[l(b)-1+b(1-2p)]^2} \\
= & \tfrac{(l(b)-1)^2 l'(b)+[2l(b)-(l(b)-1)][l(b)-1](1-2p)-b(l(b)-1)+b-b^2(1-2p)}{[l(b)-1+b(1-2p)]^2}.
\end{aligned}$$

In the derivation, I have used $l(b) = l'(b)b$. Each term in the numerator of the last expression is nonnegative, and some terms are strictly positive (for example, b). Thus $\partial(L.H.S. \text{ of } (4))/\partial b > 0$.

Now I use the Implicit Function Theorem to conclude

$$\frac{\partial \tilde{N}}{\partial b} = -\frac{\partial(L.H.S. \text{ of } (4))/\partial b}{\partial(L.H.S. \text{ of } (4))/\partial n} < 0.$$

□

Proof. (of Lemma 7, Page 20) First note that y_i is the action that maximizes the decision maker's expected utility given that m_i is received, and hence must satisfy the relevant first order conditions. Thus although y_i depends on p, I may use the Envelope Theorem and ignore such dependence when taking derivatives of V^D with respect to p. Therefore

$$\begin{aligned}
-\frac{\partial V_N^D(P,B)}{\partial p} = & \textstyle\sum_{i=1}^{n-1} p[(a_i^b-y_i)^2-(a_i^b-y_{i+1})^2]\frac{\partial a_i^b}{\partial p} + (1-p)[(a_i^{-b}-y_i)^2-(a_i^{-b}-y_{i+1})^2]\frac{\partial a_i^{-b}}{\partial p} \\
& + \textstyle\sum_{i=1}^{n} \int_{a_{i-1}^b}^{a_i^b} (s-y_i)^2 ds - \int_{a_{i-1}^{-b}}^{a_i^{-b}} (s-y_i)^2 ds. \qquad (12)
\end{aligned}$$

I evaluate the two terms on the right hand side separately. Denote the first term by A_1 and the second term A_2.

First I calculate A_1. By the definitions of a_i^β and a_i, I have for all $i = 1, ..., n-1$,

$$\begin{aligned}
a_i^b + a_i^{-b} - y_i - y_{i+1} = \; & 2a_i - (y_i + y_{i+1}) & = 0, \\
\partial a_i^b/\partial p = \; & \partial a_i^{-b}/\partial p & = \partial a_i/\partial p.
\end{aligned}$$

Therefore

$$A_1 = \sum_{i=1}^{n-1} (2p-1)[(a_i-b-y_i)^2-(a_i-b-y_{i+1})^2]\frac{\partial a_i}{\partial p}.$$

Note $(a_i - b - y_i) + (a_i - b - y_{i+1}) = -2b < 0$ and $y_i < a_i < y_{i+1}$. Therefore $|a_i - b - y_i| < |a_i - b - y_{i+1}|$. Since $p \geq \frac{1}{2}$ and $\frac{\partial a_i}{\partial p} < 0$ by Corollary 1.1, $A_1 \geq 0$ with equality sign only at $p = \frac{1}{2}$.

Now I consider A_2. It is useful to consider the first and the last terms of the summation ($i = 1$ and $i = n$) separately from the others. I have

$$\begin{aligned}
& \int_0^{a_1-b}(s-y_1)^2 ds - \int_0^{a_1+b}(s-y_1)^2 ds + \int_{a_{n-1}-b}^{1}(s-y_n)^2 ds - \int_{a_{n-1}+b}^{1}(s-y_n)^2 ds \\
= & \quad \tfrac{1}{3}[(a_1-b-y_1)^3 - (a_1+b-y_1)^3 - (a_{n-1}-b-y_n)^3 + (a_{n-1}+b-y_n)^3] \\
= & \quad \tfrac{1}{3}[-2\cdot 3(a_1-y_1)^2 b + 2\cdot 3(a_{n-1}-y_n)^2 b] \\
= & \quad \tfrac{b}{2}(-\delta(a_1)^2 + \delta(a_{n-1}-1)^2).
\end{aligned}$$

The last step uses equation (3). Now using equations (7) and (8), I conclude

$$\frac{b}{2}(-\delta(a_1)^2 + \delta(a_{n-1}-1)^2) \quad = \tfrac{b}{2}\cdot 2(a_1 - a_{n-1})\cdot 4b(1-2p) \geq 0.$$

The inequality sign is because $a_1 \leq a_{n-1}$ and $1 - 2p \leq 0$. Thus the sum of the first and last summation terms in $A2$ is nonnegative. Note that it holds as equality only if $p = \frac{1}{2}$ or $n = 2$.

The other terms ($i = 2, ..., n-1$) in A_2 can be calculated using (5):

$$\begin{aligned}
& \int_{a_{i-1}-b}^{a_i-b}(s-y_i)^2 ds - \int_{a_{i-1}+b}^{a_i+b}(s-y_i)^2 ds \\
= & \quad \tfrac{1}{3}[(a_i-b-y_i)^3 - (a_{i-1}-b-y_i)^3 + (a_i+b-y_i)^3 - (a_{i-1}+b-y_i)^3] \\
= & \quad \tfrac{1}{3}[2(a_i-y_i)^3 + 2\cdot 3(a_i-y_i)b^2 - 2(a_{i-1}-y_i)^3 - 2\cdot 3(a_{i-1}-y_i)b^2] \\
\geq & \quad 0.
\end{aligned}$$

The last inequality simply uses the fact $a_i \geq a_{i-1}$. Thus A_2 is nonnegative, and equals zero only if $p = \frac{1}{2}$ or $n = 2$.

To conclude, $\frac{\partial V_N^D(P,B)}{\partial p} \leq 0$ and is equal to zero only if $p = \frac{1}{2}$. □

Proof. (of Theorem 2, Page 20) The argument is simple. Lemma 7 states that for the same partition size, the decision maker always strictly prefer the equilibrium with smaller p. On the other hand, Corollary 1.2 implies that when p is smaller, there is also a (weakly) larger set of partition sizes to choose from since $N(p, b)$ is nonincreasing in p. Thus the most informative equilibrium with smaller $p \in [\frac{1}{2}, 1]$ always gives the decision maker strictly higher payoffs. □

The following lemma compares boundary points of size-n and size-$n+1$ equilibria. It is useful in the proof of Lemma 8.

Lemma 12. *If both size-n and size-$n+1$ equilibria exist, then for $i = 1, ..., n-1$,*

1. $a_i(n+1) < a_i(n) < a_{i+1}(n+1)$.
2. $a_{i+1}(n+1) - a_i(n+1) < a_i(n) - a_{i-1}(n)$.

Proof. **Part 1.** I show $a_i(n+1) < a_i(n)$ by using the forward version of (5). The proof is by induction.

First I show $a_1(n+1) < a_1(n)$. Suppose to the contrary, $a_1(n+1) \geq a_1(n)$. From (5), I get

$$a_i - a_{i-1} = \delta(a_1, p, b) - 2b(1-2p) - 4b(i-2)(1-2p).$$

By Fact 1, $\delta_a > 0$. Thus if $a_1(n+1) \geq a_1(n)$, I have

$$a_i(n+1) - a_{i-1}(n+1) \geq a_i(n) - a_{i-1}(n),$$

which implies that

$$a_i(n+1) \geq a_i(n)$$

for $i = 1, ..., n-1$. Since $a_n(n+1) - a_{n-1}(n+1) = a_{n-1}(n+1) - a_{n-2}(n+1) - 4b(1-2p) \geq a_{n-1}(n+1) - a_{n-2}(n+1)$, I have

$$a_n(n+1) - a_{n-1}(n+1) \geq a_{n-1}(n) - a_{n-2}(n). \tag{13}$$

Observe that whenever $a_1 > b$ or $p \neq \frac{1}{2}$, $\delta(a_1) - 2b(1-2p)$ must be strictly positive. But for finite n, $a_1 = b$ and $p = \frac{1}{2}$ cannot both be true. So

$$a_n(n+1) - a_{n-1}(n+1) = \delta(a_1(n+1)) - 2b(1-2p) - 4b(n+1-2)(1-2p) > 0.$$

Thus $a_n(n+1) > a_{n-1}(n+1) \geq a_{n-1}(n)$. Fact 1 says $\delta_a > 0$. Therefore

$$-\delta(a_n(n+1) - 1, \cdot) + 2b(1-2p) < -\delta(a_{n-1}(n) - 1, \cdot) + 2b(1-2p). \tag{14}$$

Now since $\{a_i(n)\}$ and $\{a_i(n+1)\}$ are equilibrium boundary points, they must satisfy the following conditions:

$$-\delta(a_{n-1}(n) - 1, \cdot) + 2b(1-2p) = a_{n-1}(n) - a_{n-2}(n), \tag{15}$$

$$-\delta(a_n(n+1) - 1, \cdot) + 2b(1-2p) = a_n(n+1) - a_{n-1}(n+1). \tag{16}$$

These contradict (13) and (14).

Now I perform the second step of the induction. Assume $a_j(n+1) < a_j(n)$ for $j = 1, ..., i-1$. I want to show that $a_i(n+1) < a_i(n)$.

Suppose to the contrary, $a_i(n+1) \geq a_i(n)$. This implies that

$$a_i(n+1) - a_{i-1}(n+1) > a_i(n) - a_{i-1}(n)$$

by the induction hypothesis. By (5),

$$a_j(n+1) - a_{j-1}(n+1) > a_j(n) - a_{j-1}(n)$$

for $j = i, ..., n-1$. As in the first step, I can derive (13) and (14), which again contradict (15) and (16).

That $a_i(n) < a_{i+1}(n+1)$ can be similarly proved by using the backward version of (5).

Part 2. That $a_{i+1}(n+1) - a_i(n+1) < a_i(n) - a_{i-1}(n)$ is an immediate consequence of Part 1. To see this, suppose $a_{i+1}(n+1) - a_i(n+1) \geq a_i(n) - a_{i-1}(n)$ instead. This would imply $a_n(n+1) - a_{n-1}(n+1) \geq a_{n-1}(n) - a_{n-2}(n)$. But $a_n(n+1) > a_{n-1}(n)$. This leads to a contradiction similar to that in the proof of Part 1.

Lemma 12 is thus proved. □

Proof. (of Lemma 8, Page 21) Let V_n^D and V_{n+1}^D be the decision maker's expected utility from the size-n and size-$n+1$ equilibria, respectively. The goal is to show $V_n^D < V_{n+1}^D$.

I show that the equilibrium with $n+1$ partition units can be deformed into a refinement of the equilibrium with n partition units. Let $\{a_i(n)\}_{i=0}^n$ and $\{a_i(n+1)\}_{i=0}^{n+1}$ be the boundary points that respectively correspond to the size-n and size-$n+1$ equilibria. Thus they must satisfy (5) for their respective sizes.

By Lemma 12, $[a_{n-1}(n), a_n(n+1)]$ is not degenerate. Thus choose any $a_n^x(n+1) = x \in [a_{n-1}(n), a_n(n+1)]$. I construct a partial solution $\left\{a_{n+1-i}^x\right\}_{i=0}^{n+1}$ to the backward version of (5) of size-$n+1$. For notational convenience, I do not explicitly refer to the size $n+1$ below. Of course $a_{n+1}^x = 1$ and $a_0^x = 0$. The solution is "partial" in the sense that given the initial value a_n^x, a_{n+1-i}^x is determined according to the backward version of (9) for $i = 2, ..., n-1$. However, there exists a_1^x satisfying both

$$a_1^x - a_2^x = a_2^x - a_3^x - 4b(1-2p)$$

and

$$-\delta(a_1^x) + 2b(1-2p) = a_1^x - a_2^x \tag{17}$$

only if $a_n^x = x = a_n(n+1)$ due to the uniqueness of the size-$n+1$ equilibrium.

I choose a_1^x to satisfy (17). First I show such a_1^x exists.

Claim. For each $x \in [a_{n-1}(n), a_n(n+1)]$, there exists $a_1^x \in [b, a_2^x)$ such that $-\delta(a_1^x) + 2b(1-2p) = a_1^x - a_2^x$.

Proof of Claim. Note that $a_{n+1-i}^x = \lambda(a_n^x - 1, i)$ for $i = 1, ..., n-1$ by the definition of λ. Since $\lambda_a > 0$ by Fact 2, for each $x \in [a_{n-1}(n), a_n(n+1)]$ and $i = 1, ..., n-1$, I have $a_{n+1-i}^x \in [a_{n-i}(n), a_{n+1-i}(n+1)]$ is increasing in x. The intervals are nonempty by Lemma 12. In particular, $a_2^x \in [a_1(n), a_2(n+1)]$.

First I set $a_1^x = a_2^x$. I have $-\delta(a_1^x) < 0$, since $b \leq a_1(n+1) < a_1(n) \leq a_2^x$. Note $a_1(n+1) < a_1(n)$ is implied by Lemma 12. So the L.H.S. of (17) is negative. But the R.H.S. of (17) is 0. So

$$\text{L.H.S. of (17)} < \text{R.H.S. of (17)}$$

at $a_1^x = a_2^x$.

Now if I can show

$$\text{L.H.S. of (17)} \geq \text{R.H.S. of (17)}$$

at $a_1^x = b$, then the desired statement follows from the Intermediate Value Theorem since both sides of (17) are continuous in a_1^x.

If I set $a_1^x = b$, then the L.H.S. of (17) is $2b(1-2p) \leq 0$ if $p < 1$, and $-4b$ if $p = 1$.[36] The R.H.S. of (17) is $b - a_2^x$. Since $a_2^x \geq a_1(n)$ it suffices to show $2b(1-2p) \geq b - a_1(n)$ when $p \neq 1$, and $-4b \geq b - a_1(n)$ for $p = 1$.

When $p \neq 1$, suppose to the contrary that $a_1(n) < b - 2b(1-2p)$. Then since $\delta_a > 0$, $a_2(n) - a_1(n) = \delta(a_1(n)) < \delta(b - 2b(1-2p), \cdot) = -4b(1-2p)$. The last equality sign is obtained by substituting $a = b - 2b(1-2p)$ into the definition of δ. On the other hand, $a_3(n+1) - a_2(n+1) = a_2(n+1) - a_1(n+1) - 4b(1-2p) \geq -4b(1-2p)$. This violates Part 2 of Lemma 12.

When $p = 1$, suppose to the contrary that $a_1(n) < b + 4b$. Then $\delta(a_1(n)) = a_1 + b < 6b$, and thus $a_2(n) - a_1(n) = \delta(a_1(n)) < 6b$. On the other hand, $a_3(n+1) - a_2(n+1) = a_2(n+1) - a_1(n+1) - 4b(1-2p) = \delta(a_1(n+1)) + 4b = a_1(n+1) + b + 4b \geq 6b$. This again violates Part 2 of Lemma 12.

The claim is thus proved. $\square$

By applying the Implicit Function Theorem, it is straightforward to show a_1^x is strictly increasing in a_2^x. Furthermore, since a_2^x is strictly increasing in x by Lemma 10 and all terms in (17) depend on x through a_2^x, a_1^x is strictly increasing in x.

Let $\hat{a}_1^x$ be determined by $\hat{a}_1^x - a_2^x = a_2^x - a_3^x - 4b(1-2p)$. Note that $a_1^x = \hat{a}_1^x$ for $x = a_n(n+1)$. On the other hand, for any $x \in [a_{n-1}(n), a_n(n+1))$, $\hat{a}_1^x - a_2^x <$

[36] Recall $\delta(a_1, p, b) = a_1 + b$ for $p = 1$.

$a_1(n+1)-a_2(n+1)$ and $\hat{a}_1^x < a_1(n+1)$ by Lemma 10. Since $-\delta(a_1(n+1))+2b(1-2p) = a_1(n+1)-a_2(n+1)$, Fact 1 implies that $-\delta(\hat{a}_1^x)+2b(1-2p) > \hat{a}_1^x - a_2^x$. Thus $a_1^x > \hat{a}_1^x$ for $x \in [a_{n-1}(n), a_n(n+1))$.

Now let me interpret the partition $\{a_i^x\}_{i=0}^{n+1}$ as the following strategy profile:[37]

1. Message m_i is sent when the expert's bias is β and $s \in [a_{i-1}^{x,\beta}, a_i^{x,\beta}]$ for $i = 1, ..., n+1$, where $a_0^{x,\beta} = 0$, $a_{n+1}^{x,\beta} = 1$, and $a_i^{x,\beta} = a_i^x - \beta$ for $i = 1, ..., n$.

2. When receiving the message m_i, the decision maker forms Bayesian beliefs, and chooses action y_i^x that is equal to the expected value of s conditional on m_i being sent. That is, $\{y_i^x\}$ and $\{a_i^x\}$ satisfy the size-$n+1$ version of 3.

Let $V^D(x)$ be the decision maker's expected utility from the above strategy profile.

First, $\{a_i^x\}_{i=0}^{n+1}$ is the same as $\{a_i(n)\}_{i=0}^{n+1}$ when $x = a_n(n+1)$. So $V^D(a_n(n+1)) = V_{n+1}^D$. Second, when $x = a_{n-1}(n)$, $a_2^x = a_1(n)$. Thus $\{a_i^x\}_{i=0}^{n+1}$ is a refinement of the size-n equilibrium $\{a_i(n)\}_{i=0}^{n}$ at $x = a_{n-1}(n)$. So $V^D(a_{n-1}(n)) \geq V_{n-1}^D$. Now to prove $V_{n+1}^D > V_n^D$, it suffices to show $V^D(x)$ is strictly increasing in x, which implies $V^D(a_n(n+1)) > V^D(a_{n-1}(n))$.

Now

$$-V^D(x) = p\sum_{i=1}^{n+1}\int_{a_{i-1}^{x,b}}^{a_i^{x,b}} (s-y_i^x)^2\, ds + (1-p)\sum_{i=1}^{n+1}\int_{a_{i-1}^{x,-b}}^{a_i^{x,-b}} (s-y_i^x)^2\, .ds$$

Applying the Envelope Theorem, I may ignore the indirect effect x has on V^D through the $\{y_i^x\}$, since the $\{y_i^x\}$ are chosen to maximize V^D. So

$$\begin{aligned} -\frac{dV^D(x)}{dx} &= p\sum_{i=1}^{n}[(a_i^{x,b}-y_i^x)^2 - (a_i^{x,b}-y_{i+1}^x)^2]\frac{\partial a_i^{x,b}}{\partial x} + \\ &\quad (1-p)\sum_{i=1}^{n}[(a_i^{x,-b}-y_i^x)^2 - (a_i^{x,-b}-y_{i+1}^x)^2]\frac{\partial a_i^{x,-b}}{\partial x}. \end{aligned}$$

By definition $\frac{\partial a_i^{x,b}}{\partial x} = \frac{\partial a_i^{x,-b}}{\partial x} = \frac{\partial a_i^x}{\partial x}$. Furthermore, by Lemma 10, $\frac{\partial a_i^x}{\partial x} > 0$. Therefore, to prove $\frac{dV^D(x)}{dx} > 0$, it suffices to show

$$p[(a_i^x - b - y_i^x)^2 - (a_i^x - b - y_{i+1}^x)^2] + (1-p)[(a_i^x + b - y_i^x)^2 - (a_i^x + b - y_{i+1}^x)^2] \leq 0$$

for all $i = 1, ..., n$ and strictly negative for some such i. The proof of this statement is as follows.

[37] The strategy profile does not have to be an equilibrium. The purpose is to calculate the decision maker's expected payoff, and show how it depends on the initial value x.

Note that (5) is satisfied by the $\{a_i\}$ if and only if the $\{y_i^x\}$ satisfy (3) and $a_i = \frac{y_i+y_{i+1}}{2}$. Observe that a_i^x satisfies all equations of the backward size-$n+1$ version of (5) but the last two, and that the $\{y_i^x\}$ are defined so as to satisfy the size-$n+1$ version of (3). I may then conclude that $a_i^x = \frac{y_i^x+y_{i+1}^x}{2}$ for $i = 3, ..., n$.[38] This implies that for $i = 3, ..., n$,

$$\begin{aligned} a_i^x - b - y_i^x &= -(a_i^x + b - y_{i+1}^x) \\ a_i^x - b - y_{i+1}^x &= -(a_i^x + b - y_i^x) \\ (a_i^x - b - y_i^x) + (a_i^x - b - y_{i+1}^x) &= -2b. \end{aligned}$$

The last equality then implies $|a_i^x - b - y_i^x| < \left|a_i^x - b - y_{i+1}^x\right|$. Thus for $i = 3, ..., n$,

$$\begin{aligned} & p[(a_i^x - b - y_i^x)^2 - (a_i^x - b - y_{i+1}^x)^2] + (1-p)[(a_i^x + b - y_i^x)^2 - (a_i^x + b - y_{i+1}^x)^2] \\ = & \quad (2p-1)[(a_i^x - b - y_i^x)^2 - (a_i^x - b - y_{i+1}^x)^2] \\ \leq & \quad 0. \end{aligned}$$

Now I consider $i = 2$ and $i = 1$. Observe that $y_2^x = \frac{a_1^x + a_2^x}{2} + b(1-2p)$. Now let $\hat{y}_2^x = \frac{\hat{a}_1^x + a_2^x}{2} + b(1-2p)$. Note that $y_2^x - \hat{y}_2^x = \frac{a_1^x - \hat{a}_1^x}{2} < 0$. Since $\hat{a}_1^x$ satisfies the second to last equation of the backward version of 5, $a_2^x = \frac{\hat{y}_2^x + y_3^x}{2}$ holds. Thus

$$\begin{aligned} a_2^x - b - \hat{y}_2^x &= -(a_2^x + b - y_3^x) \\ a_2^x - b - y_3^x &= -(a_2^x + b - y_2^x) \\ (a_2^x - b - \hat{y}_2^x) + (a_2^x - b - y_3^x) &= -2b. \end{aligned}$$

Similarly to the above, $|a_i^x - b - y_i^x| < \left|a_i^x - b - y_{i+1}^x\right|$. Now I use the identity $y_2^x = \hat{y}_2^x + (y_2^x - \hat{y}_2^x)$, and get

$$\begin{aligned} & p[(a_2^x - b - y_2^x)^2 - (a_2^x - b - y_3^x)^2] + (1-p)[(a_2^x + b - y_2^x)^2 - (a_2^x + b - y_3)^2] \\ = & \quad (2p-1)[(a_2^x - b - \hat{y}_2)^2 - (a_2^x - b - y_3)^2] + [2(a_2^x - \hat{y}_2^x + b(1-2p))(\hat{y}_2^x - y_2^x) + (\hat{y}_2^x - y_2^x)^2] \end{aligned}$$

The first term in the last expression is nonpositive by arguments similar to those used similarly to the case $i = 3, ..., n$. Now I consider the second term. Since $\hat{y}_2^x = \frac{\hat{a}_1^x + a_2^x}{2} + b(1-2p)$, I have $a_2^x - \hat{y}_2^x + b(1-2p) = \frac{a_2^x - \hat{a}_1^x}{2} > 0$. On the other hand, $0 > \hat{y}_2^x - y_2^x = \frac{\hat{a}_1^x - a_1^x}{2} > \frac{\hat{a}_1^x - a_2^x}{2}$ by the relationship $a_2^x > a_1^x > \hat{a}_1^x$. So $2(a_2^x - \hat{y}_2^x + b(1-2p))(\hat{y}_2^x - y_2^x) + (\hat{y}_2^x - y_2^x)^2 < 0$. I then have

$$p[(a_2^x - b - y_2^x)^2 - (a_2^x - b - y_3^x)^2] + (1-p)[(a_2^x + b - y_2^x)^2 - (a_2^x + b - y_3)^2] < 0.$$

[38] The details of the argument are available from the author.

Since a_1^x is chosen to satisfy (17), or the first equation in (5), I have $a_1^x = \frac{y_1^x + y_2^x}{2}$. Thus as in the $i = 3, ..., n$ case,

$$p[(a_1^x - b - y_1^x)^2 - (a_1^x - b - y_2^x)^2] + (1-p)[(a_1^x + b - y_1^x)^2 - (a_1^x + b - y_2^x)^2] \leq 0.$$

Summarizing the argument above,

$$p[(a_i^x - b - y_i^x)^2 - (a_i^x - b - y_{i+1}^x)^2] + (1-p)[(a_i^x + b - y_i^x)^2 - (a_i^x + b - y_{i+1}^x)^2] \leq 0$$

for all $i = 1, ..., n$ and strictly negative for $i = 2$.

Thus I conclude $\frac{dV^D(x)}{dx} > 0$. Hence $V_{n+1}^D > V^D(a_{n-1}(n)) \geq V_n^D$. □

Proof. (of Lemma 9) As in the proof of Lemma 7, I may use the Envelope Theorem and ignore the indirect dependence of V^D on b through y_i. Therefore

$$-\frac{\partial V_n^D(p,b)}{\partial b} = \sum_{i=1}^{n-1} p[(a_i^b - y_i)^2 - (a_i^b - y_{i+1})^2]\frac{\partial a_i^b}{\partial b} + (1-p)[(a_i^{-b} - y_i)^2 - (a_i^{-b} - y_{i+1})^2]\frac{\partial a_i^{-b}}{\partial b}.$$

Using the definition $a_i^\beta = a_i - \beta$ for $i = 1, ..., n$, and the fact that $a_i = \frac{y_i + y_{i+1}}{2}$, I have

$$-\frac{\partial V^D}{\partial b} = \sum_{i=1}^{n-1} [(a_i^b - y_i)^2 - (a_i^b - y_{i+1})^2][(2p-1)\frac{\partial a_i}{\partial b} - 1]. \tag{18}$$

I know from the proof of Lemma 7 that $[(a_i^b - y_i)^2 - (a_i^b - y_{i+1})^2] < 0$ for $i = 1, ..., n$. Now I show that $(2p-1)\frac{\partial a_i}{\partial b} - 1 < 0$. This implies that $\frac{\partial V^D}{\partial b} < 0$.

Let $\alpha_i \equiv pa_i^b - (1-p)a_i^{-b}$.[39] Note that $\alpha_i = (2p-1)a_i - b$ for $i = 1, ..., n-1$. The task is thus to show $\frac{\partial \alpha_i}{\partial b} < 0$ for $i = 1, ..., n-1$. I may rewrite (5) as

$$\begin{aligned}
\alpha_2 - \alpha_1 &= \alpha_1 + b + b \cdot \frac{\alpha_1}{\alpha_1 + 4p(1-p)b} + 2b(1-2p)^2 \\
\alpha_i - \alpha_{i-1} &= \alpha_{i-1} - \alpha_{i-2} + 4b(1-2p)^2 \qquad i = 3, ..., n-1, \\
\alpha_{n-1} - \alpha_{n-2} &= -[\alpha_{n-1} - (2p-1) + b + b \cdot \frac{\alpha_{n-1} - (2p-1)}{\alpha_{n-1} + 4p(1-p)b - (2p-1)} + 2b(1-2p)^2].
\end{aligned} \tag{19}$$

Considering all equations but the last, I may establish a result similar to Lemma 10. That is

1. $\alpha_i - \alpha_{i-1}$ is strictly increasing in α_1 and b for all $\alpha_1 \geq (2p-1)b - b$ and $i = 2, ..., n-1$.

2. α_i is also strictly increasing in α_1 and b for $i = 2, ..., n-1$.

[39]The symbol α_i since it has the CS model as the special case when $p = 1$, as in Section 3.

The proof is similar to that of Lemma 10 and is omitted here.

Now consider the last equation of (19). The L.H.S. is increasing in a_1 and b. The R.H.S. is strictly decreasing in a_{n-1}, since

$$\frac{\partial \frac{\alpha_{n-1}-(2p-1)}{\alpha_{n-1}+4p(1-p)b-(2p-1)}}{\partial a_{n-1}} = \frac{4p(1-p)b}{[a_{n-1}+4p(1-p)b-(2p-1)]^2} \geq 0.$$

Hence the R.H.S. is strictly decreasing in a_1 since a_{n-1} is strictly increasing in a_1 and a_{n-1} is the only channel through which the R.H.S. depends on a_1. The R.H.S. is strictly decreasing in b, since

$$\frac{\partial \frac{b[\alpha_{n-1}-(2p-1)]}{\alpha_{n-1}+4p(1-p)b-(2p-1)}}{\partial a_{n-1}} = \frac{[a_{n-1}-(2p-1)]^2+4p(1-p)b^2\frac{\partial a_{n-1}}{\partial b}}{[a_{n-1}+4p(1-p)b-(2p-1)]^2}$$

and $\frac{\partial a_{n-1}}{\partial b} > 0$. The Implicit Function Theorem thus implies that in equilibrium, a_1 decreases as b increases. That a_i $(i = 2, ..., n-1)$ is strictly decreasing in b can be shown by induction, as in the proof of Corollary 1.1.

Therefore, fixing partition size n and prior p,

$$\frac{\partial V_n^D(p,b)}{\partial b} < 0.$$

$\square$

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